CHAPTER 3. CAPACITY OF SIGNALIZED INTERSECTIONS

1. Overview

In this chapter we explore the models on which the HCM capacity analysis method for signalized intersections are based. While the method has increased in complexity since the publication of the first version in the 1950 HCM, the basic concept remains the same: the capacity of an approach or movement is the product of the saturation flow rate and the proportion of the hour that is green for that approach or movement.

Table 1 shows a roadmap to the material presented in this chapter. The first five sections provided an overview. We start in section 2 with a discussion of how signalized intersections operate in the field, focusing on the operation of one approach. In section 3, we define traffic movements and signal phases, both important in describing the operation of signalized intersections. Signal control, both pretimed and actuated, is described in section 4. In section 5, we identify the important factors in the interaction of traffic flow and signal control that will help us to formulate the models to predict the capacity of an intersection approach.

Table 1. Roadmap to chapter 3

Overview	Simplified Scenarios	Learning in Depth	Closing
1. Overview 2. What do we observe in the field 3. Movements and phases 4. Actuated control processes 5. Formulating the model	6. Capacity of one approach 7. Capacity utilization for intersection 8. Delay for approach 9. Delay for approach 10. Capacity of one lane 11. Delay for non-uniform arrivals 12. Predicted green time for actuated control	13. Calculating saturation headways 14. Building the computational engines and exploring the model	15. Summary 16. Glossary 17. References Appendix. Visual Basic code

The models that we formulate in sections 6 through 12 are based on seven “simplified scenarios”, scenarios in which only the most important and relevant traffic, geometric, and control factors are considered. By focusing only on these factors, you will develop a basic understanding of the operation of a signalized intersection, one that you can later build upon as the more complex conditions found in the real world are considered.

Table 2 shows the attributes of each scenario. The scenarios include:

- Calculating the capacity of an approach based on one signal cycle.
- Calculating the capacity utilization for an intersection using critical movement analysis.
- Calculating the uniform delay of an approach when demand is less than capacity.
- Calculating the delay when demand exceeds capacity.
- Calculating the capacity of an exclusive left turn lane for permitted LT movements.
- Calculating delay when the arrival pattern is non-uniform.
- Predicting average green time for a phase under actuated control.

Table 2. Attributes of simplified scenarios

Scenario Description	Signal Control	Demand and Capacity	LT Operation	Arrival Pattern	Movements Considered	Facility
1. Capacity of one approach	P	d<c	None	Uniform	TH	Approach
2. Capacity utilization for intersection	P	d<c	Protected, permitted	Uniform	LT, TH	Intersection
3. Delay for approach	P	d<c	None	Uniform	TH	Approach
4. Delay for approach	P	d>c	None	Uniform	TH	Approach
5. Capacity of one lane	P	d<c	Permitted	Uniform	LT, TH	Approach
6. Delay for non-uniform arrivals	P	d<c	None	Non-uniform	TH	Approach
7. Predicted green time for actuated control	A	d<c	None	Uniform	TH	Intersection

Notes: P = pretimed, A = actuated, d = demand, c = capacity, LT = left turn, TH = through.

You will see that the operation of a signalized intersection is based on the decisions made by drivers as they interact with the traffic control system. Saturation headway is the primary model parameter on which capacity prediction is based. You will learn how to calculate the saturation headway in Section 13.

You will build a series of computational engines in section 14, based on four of the scenarios described above. You can use these computational engines (in the form of spreadsheets) to study the predictions of capacity that the models make under a range of traffic flow and signal control conditions. These predictions help you to understand when signal control might be an effective control strategy and under what conditions other types of control should be considered.

Section 15 presents a summary of the chapter. Section 16 presents a list of all terms and variables used in the chapter and provides a definition or description for each. References are presented in section 17.

2. What Do We Observe in the Field?

A signalized intersection controls the flow of traffic by showing a green indication only to those movements that can safely travel through the intersection together at the same time. A time separation, indicated by the yellow and red displays, is provided between each set of these compatible movements.

Since we are interested in predicting the capacity of a lane or approach, let's consider what drivers see and do as they approach a signalized intersection. Some drivers arrive at the intersection when the signal indication is red. In this case, they come to a stop and join the other vehicles waiting in the queue. When the signalized indication changes to green, the queue begins to move as drivers get up to speed and enter the intersection. The queue eventually clears.

Other drivers arrive at the intersection when a green indication is displayed. If the queue that formed during red is still clearing, these drivers join the queue and as it clears, enter the intersection. If the queue has already cleared, drivers simply travel through the intersection without slowing or stopping.

So when drivers are given the green indication, they can enter the intersection; otherwise, if the indication is red, they must wait. The capacity of a lane or approach thus depends on two primary factors: (1) the maximum flow rate at which drivers can pass the stop line and enter the intersection and (2) the proportion of time that the signalized intersection is green for the lane or approach.

Other factors also limit the capacity of a lane or approach, such as pedestrians crossing the street or heavy vehicles that are slower to accelerate than passenger cars. But while these factors are often present in the field, we will only consider simplified scenarios in which there are no pedestrians and a traffic stream that consists only of passenger cars.

Why do we study the capacity of a signalized intersection? Sometimes, we want to determine what type of control is optimal for an intersection given demand and geometric conditions. We might also want to know when it is justified to change from stop sign control to signal control. Or, perhaps we want to know the effect of changing the signal timing or the geometric configuration of the intersection. The models that we will develop in this chapter help to answer these questions.

3. Movements and Phases

In the previous section, we considered traffic flow on one approach of a signalized intersection. In this section, we consider traffic flow at the entire intersection and how the movements on the individual approaches are sequenced and controlled. Safety and efficiency are the two primary goals of intersection control. Considering efficiency, as many movements as possible should be served concurrently. But for safety reasons, conflicting movements must be served during different periods of the signal cycle, separated in time by the yellow and red clearance intervals.

Movements

A standard intersection with four approaches can have up to twelve vehicular movements. These movements, and the way in which they are typically numbered and referred to, are shown in Figure 1.

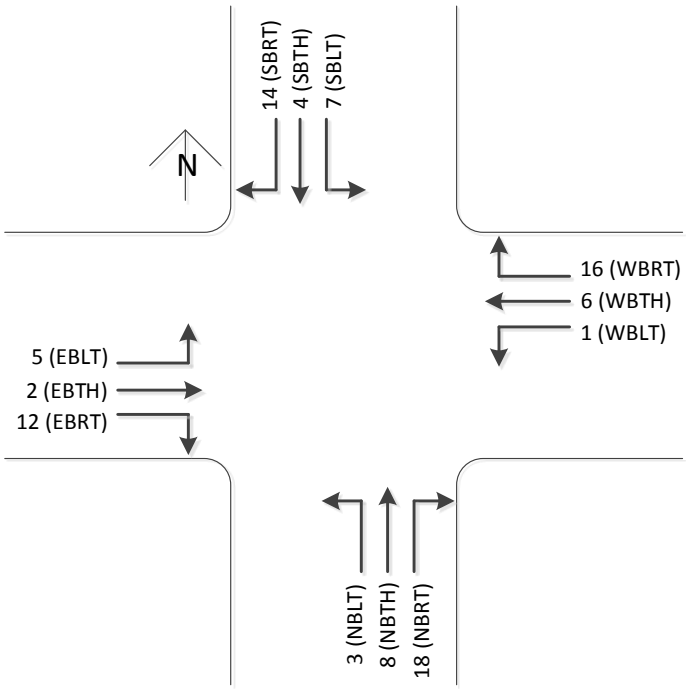


Figure 1. Numbering and notation of movements at a signalized intersection

A *movement* is defined by the direction of travel from its origin and the turning maneuver that a vehicle completes to get to its destination. For example, movement 2 begins traveling in the eastbound direction and continues through the intersection in the eastbound direction. It is referred to as an eastbound through movement (abbreviated as EBTH). Or, movement 1 begins traveling in the westbound direction, making a LT and continuing in the southbound direction. Movement 1 is referred to as a westbound left turn (WBLT) movement. As a final example, movement 14 is referred to as a southbound right turn (SBRT) movement.

A movement is also categorized by any restriction that may be placed on it. There are four such categories:

- An unopposed movement is just that: there is no other movement that opposes this movement. For example, a movement on a one-way street is unopposed.

- A protected movement may have a movement that can oppose it but the signal indication gives the protected movement the right-of-way. For example, a left turn movement may be protected if the signal indication is a green arrow while the opposing traffic movement has a red indication.
- A permitted movement is allowed to travel through the intersection, but must yield if a higher priority opposing movement is present. For example, a permitted left turn may enter and travel through the intersection as long as there are no opposing through movements also desiring to travel through the intersection at the same time.
- A movement can also be prohibited, or not allowed. This restriction can be complete or in effect only during certain periods of the day. For example, left turns can be prohibited (not allowed) during peak periods, especially if the left turn movement shares a lane with a through movement.

Groups of movements are also classified as either *compatible* or *conflicting*. In general, north-south movements conflict with east-west movements. North-south movements are part of a group called a *concurrency group* since these movements may travel concurrently; similarly east-west movements are part of the east-west concurrency group. The concept of the concurrency group is illustrated in Figure 2. Depending on the signal phasing plan (*phase* is defined in the next subsection) and restrictions on the movements, a movement in the north-south concurrency group may be served at the same time as any other movement in this group. The same concept applies to the movements in the east-west concurrency group.

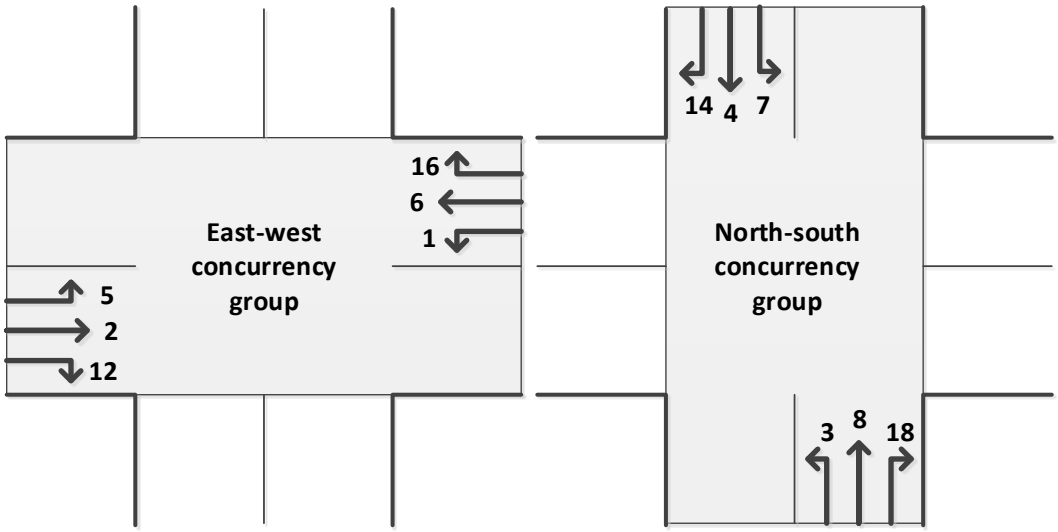


Figure 2. Concurrency groups

Phasing and the Ring Barrier Diagram

While stop and yield controlled intersections require judgment before a driver can safely enter the intersection, signal control gives an unambiguous indication whether a particular movement has the right of way or not. This right of way assignment is done through the signal display of a green ball, a green arrow, or a flashing yellow arrow.

A *phase* is a timing unit that controls one or more compatible movements at a signalized intersection. The timing unit consists of the consecutive displays of the green, yellow, and red indications shown to the movements controlled by the phase, as shown in Figure 3.

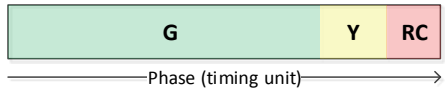


Figure 3. Phase

A *ring* is a sequence of phases that must be served one after the other. Phases are sequenced to separate conflicting movements and to make sure that those movements that are served concurrently are compatible.

A *ring barrier diagram* is the tool that is used to define those movements that are compatible and can be served concurrently, and those that conflict and must be served in sequence. The ring barrier diagram is built upon the concept of the concurrency group described above. The movements of the east-west concurrency group are served first, followed by service to the north-south concurrency group.

An example ring barrier diagram with eight phases and a two ring structure is shown in Figure 4. Barriers separate the two concurrency groups. The phase number is shown in the upper left corner of each square, and the movements that are controlled by that phase are shown with an arrow and movement number. A dashed line indicates permitted movements.

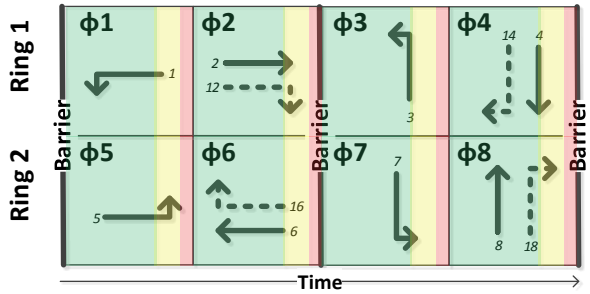


Figure 4. Ring barrier diagram for two ring, eight phase operation

The following rules apply to a ring barrier diagram:

- Each ring shows the order in which phases are sequenced.
- A phase in one ring can time concurrently with a phase in another ring as long as both phases are in the same concurrency group.
- A phase in one concurrency group cannot time concurrently with a phase in another concurrency group. (The exception to this rule is the concept of the *overlap*, which is beyond the scope of this chapter.)
- A barrier separates the north-south and the east-west concurrency groups. Both rings must cross the barrier at the same time.
- After a phase is served, change and clearance intervals (yellow indication and red indication) provide a time separation between that phase and the next conflicting phase.

Left Turn Phasing

There are several ways in which left turn movements can be served at a signalized intersection. Figure 4 showed the case of *leading left turns* (served before the through movements) that are *protected* (with no opposing through movements served at the same time). Protected left turns (shown with solid lines) are generally used when the combination of the left turn movement volume and its opposing through movement volume is high.

When left turn volumes are low, *permitted left turn* operation (shown with dashed lines) is possible. This results in a single ring two phase operation, as shown in Figure 5. Here, the left turn and through movements within each concurrency group are controlled by the same phase.

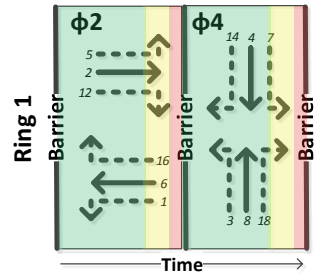


Figure 5. Two-phase ring barrier diagram

Another phase structure is called *split phasing*, in which each approach is served in sequence. Split phasing is used when safety or geometric restrictions don't allow opposing left turns to be served at the same time. This structure can be represented by a single ring, as shown in Figure 6.

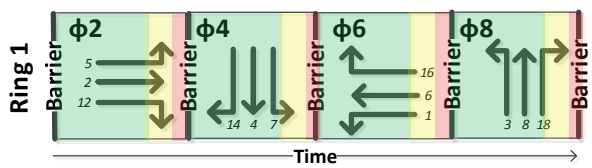


Figure 6. Split-phase ring barrier diagram

The decision whether to provide protected left turn phasing is based on the combination of the left turn and opposing through traffic volumes, the geometric layout of the intersection, the speeds of the opposing traffic, the number and kinds of traffic crashes that have occurred, and the delay and degree of queuing experienced by left turn traffic.

One common guideline is the "cross product" of the left turn volume and the sum of the opposing through and right turning volumes. The Highway Capacity Manual offers the following criterion for this guideline: the use of a protected left turn phase should be considered when, during the peak hour, the product of the left turning volume and the opposing traffic volume equals or exceeds:

- 50,000 if there is one opposing lane,
- 90,000 for two opposing lanes, and
- 110,000 for three or more opposing lanes.

Example Calculation 1. Determining Left Turn Phasing

Consider the intersection shown in Figure 7, with one or two through lanes and an exclusive left turn lane on the four approaches. The hourly flow rates for each movement are also shown in the figure.

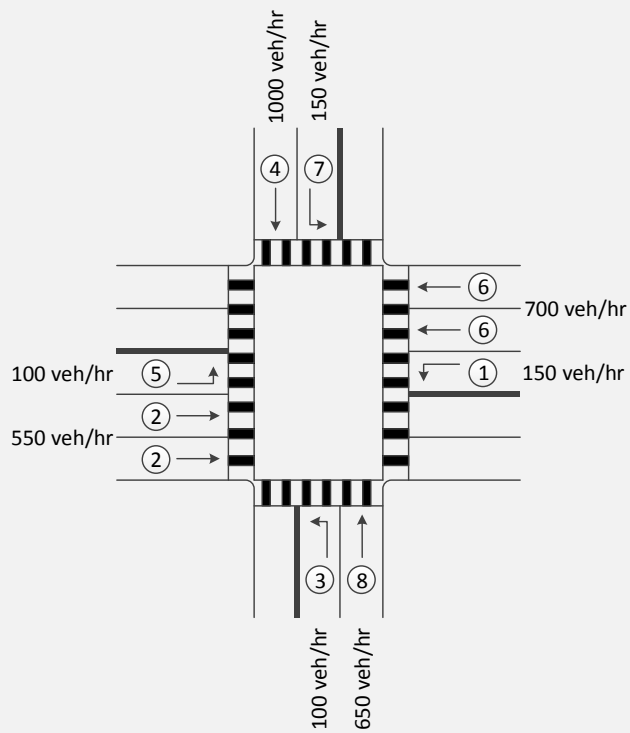


Figure 7. Flow rates for Example Calculation 1

Based on these flow rates and the intersection geometry, what left turn phasing would you recommend for each of the four approaches?

The flow rates for the left turn and through movement combinations are shown in Table 3. The cross products for each left turn-through movement combination are computed and also shown in the table. Finally, based on the criteria from the Highway Capacity Manual listed above, the recommended left turn phasing is given. In this case, the cross products are high enough for the north-south movements that protected left turn phasing would be recommended. However, permitted left turn phasing is sufficient for the east-west movements.

Table 3. LT-TH movement cross products and left turn phasing, Example Calculation 1

Movement	Flow rate (veh/hr)	Cross product	HCM guideline	Recommended LT phasing
NBLT	100	100,000	50,000	Protected
SBTH	1000			
SBLT	150	97,500	50,000	Protected
NBTH	650			
EBLT	100	70,000	90,000	Permitted
WBTH	700			
WBLT	150	82,500	90,000	Permitted
EBTH	550			

Example Calculation 2. Determining the Ring Barrier Diagram

A signalized intersection has lagging protected left turns for the NB and SB movements and permitted left turns for the EB and WB movements. Create a ring barrier diagram for this case.

Figure 8 shows the ring barrier diagram that represents the conditions described above. For the east-west concurrency group, one phase (in this case phase 2), controls all of the movements. For the north-south concurrency group, phases 4 and 8 (controlling the through and right turn movements) "lead" the left turns (controlled by phases 3 and 7). The left turns "lag" the through movements.

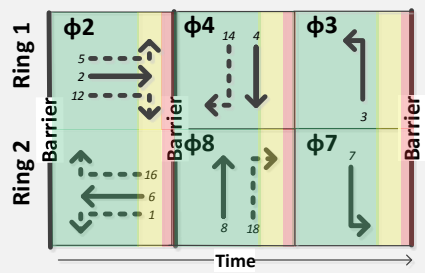


Figure 8. Ring barrier diagram for conditions in Example Calculation 2

Timing Stages

The ring-barrier concept allows compatible phases in different rings (within each concurrency group) to operate for different time durations based on the level of traffic volume. For example, if the traffic volume for the EBLT movement is greater than the volume for the WBLT movement, phase 5 (controlling the EBLT movement) can provide a longer green duration (or "time longer") than phase 1. Similarly, if the traffic volume for the NBLT movement is greater than the volume for the SBLT movement, then phase 3 can time longer than phase 7.

The conditions described above, represented in Figure 9 through the concept of the timing stage, show the intrinsic efficiency of the ring barrier process. A *timing stage* is an interval of time during which no signal displays change. The horizontal length of the phase is its relative time duration. Time moves from left to right.

- Stage 1 includes the concurrent timing of phases 1 and 5 serving the EBLT and WBLT movements. Because the volume for movement 1 is less than the volume for movement 5, phase 1 terminates before phase 5.
- The second stage is the concurrent timing of phases 2 and 5.
- When phase 5 terminates, phases 2 and 6 time concurrently in stage 3.
- The same process applies to the north-south concurrency group, shown in stages 4, 5, and 6.

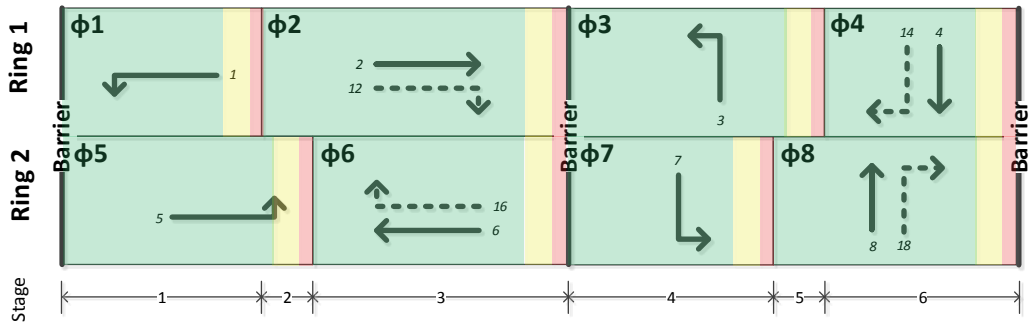


Figure 9. Ring barrier diagram and timing stages

Example Calculation 3. Calculating the Timing Stages

Consider the following timing requirements for the eight phases that serve a standard four-leg intersection. If protected left turns are required, construct a ring barrier diagram for this intersection that shows the resulting timing stages. Assume also that the left turns lead the through movements.

Table 4. Phase durations, Example Calculation 3

Phase	1	2	3	4	5	6	7	8
Duration, sec	15	30	10	25	10	35	10	25

Figure 10 shows the ring barrier diagram for leading protected left turns.

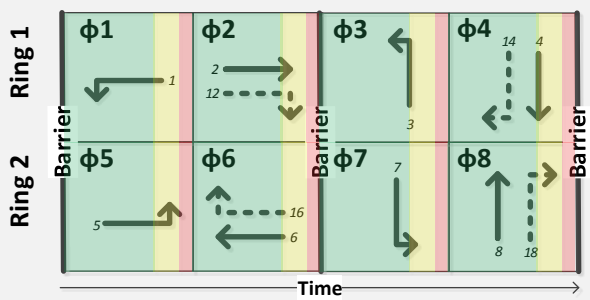


Figure 10. Ring barrier diagram, Example Calculation 3

The ring barrier diagram can also be scaled based on the time required to serve each of the phases noted by the resulting timing stages (see Figure 11).

- For the east-west concurrency group, phases 1 and 5 time concurrently for 10 sec during stage 1
- But phase 1 times for an additional 5 sec. It times concurrently with phase 6 during stage 2.
- During stage 3, phases 2 and 6 time concurrently for 30 sec.
- When the barrier is crossed, phases 3 and 7 time concurrently for 10 sec, as part of stage 4. Here both phases end at the same time.
- Stage 5 consists of phases 4 and 8 timing concurrently for 25 sec.
- Note that the sum of the durations for phases 1 through 4 is equal to the durations for phases 5 through 8.

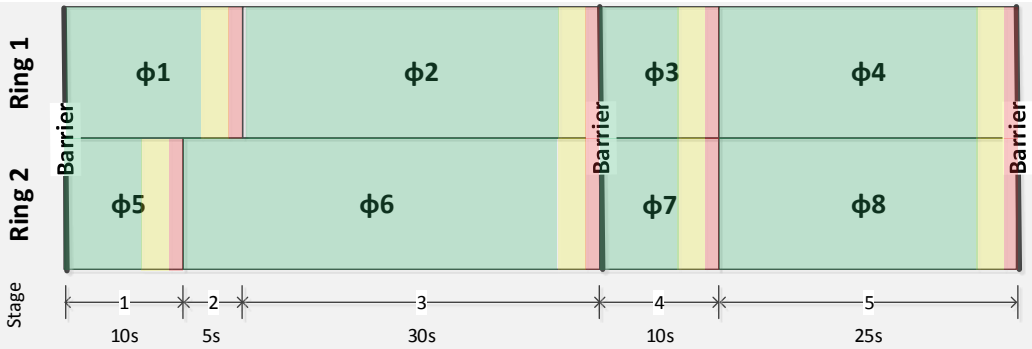


Figure 11. Ring barrier diagram with time scale

4. Actuated Signal Control Timing Processes

Traffic Control Process Diagram

Nearly all traffic signal controllers in operation today use the concept of actuated traffic control. The actuated control system includes four interdependent components: users, detectors, controllers, and displays, as represented in Figure 12. A functional representation of this system, known as a traffic control process diagram, is shown in Figure 13.

- Vehicles (users) arrive at the intersection and are detected by inductive loops buried in the pavement, video detection systems with cameras mounted over the intersection, or other sensors.
- The detector, represented in Figure 12 by a loop detector located at the stop bar, sends a call to the traffic controller when the presence of a vehicle is sensed.
- The traffic controller, based on the timing plan that has been programmed into the controller, processes the detections and determines the signals to display (green, yellow, or red in the case of vehicle displays).
- The user then responds to the signal that is displayed, and the process is complete.

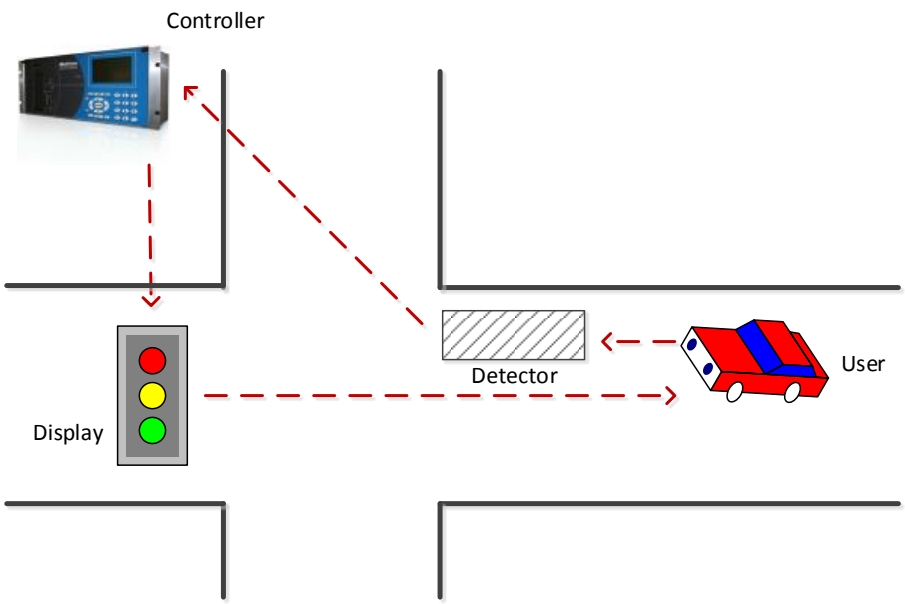


Figure 12. Components of traffic control system

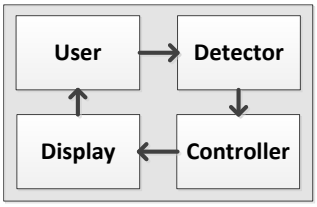


Figure 13. Traffic control process diagram

Timing Processes

There are three primary timing processes that govern the operation of the actuated traffic controller: the minimum green timer, the passage timer, and the maximum green timer.

The *minimum green time* is the minimum time that the signal display will remain green no matter what else occurs. The minimum green timer is initially set to a value equal to the minimum green time. When the phase begins timing, the minimum green timer begins to time down. The timer expires when its value reaches zero, as shown in Figure 14.

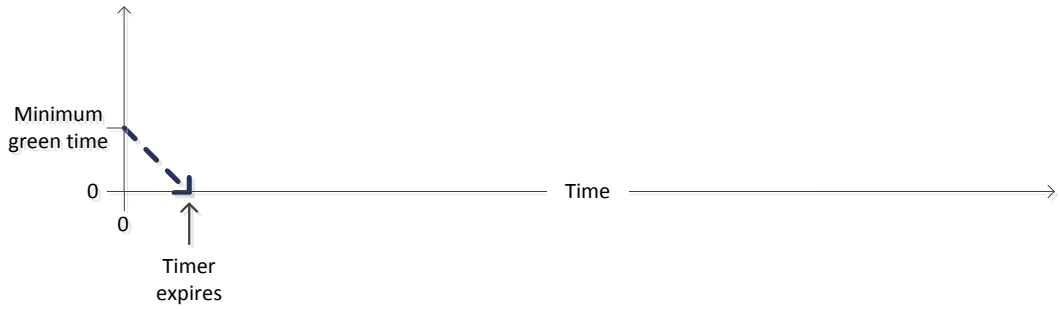


Figure 14. Minimum green timer process

The purpose of the passage timer (sometimes called the vehicle extension timer) is to extend the green display until a gap of a pre-determined size between vehicles in the traffic stream is detected. The *passage time* is the maximum time that a detector can remain unoccupied before the passage timer expires. The passage timer remains at its initial value as long as a vehicle remains in the detection zone (or, "a call is active"). Once a vehicle leaves the zone, the passage timer begins to time down. When a subsequent vehicle enters the zone, the passage timer is reset to its initial value if it hasn't already expired.

The following figures show two example timing processes for the passage timer. In Figure 15, the passage timer remains at its initial value as long as a vehicle is in the detection zone. The timer begins to time down when a vehicle leaves the detection zone. In this example, the timer expires because no subsequent vehicle enters the detection zone.

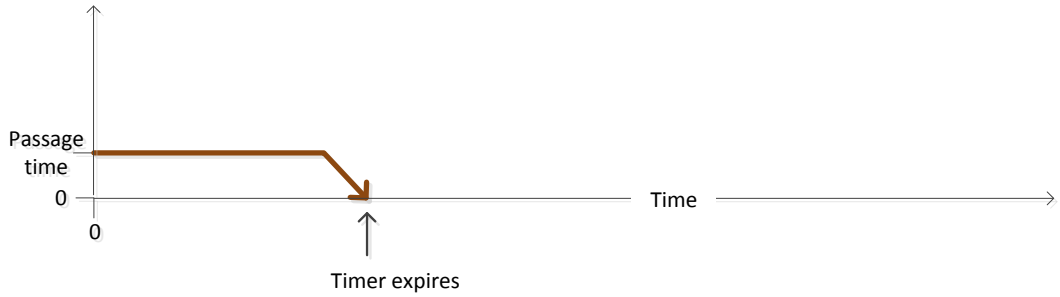


Figure 15. Passage timer process, first example

By contrast, in the second example, shown in Figure 16, the passage timer is reset several times as a vehicle leaves the zone and a subsequent vehicle arrives in the zone. This resetting occurs three times. Finally, the timer expires when no subsequent vehicle enters the detection zone and the timer reaches zero.

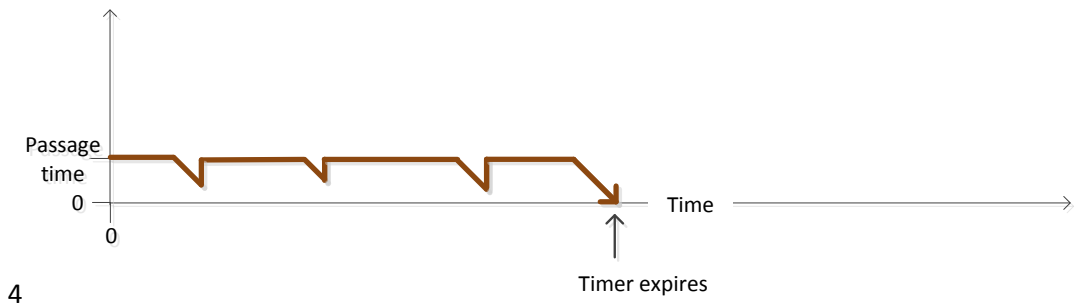


Figure 16. Passage timer process, second example

The purpose of the maximum green time is to produce a maximum cycle length that keeps delay at a reasonable level. The *maximum green time* is the maximum duration that the signal display will remain green after a call has been received on a conflicting phase. When such a call is received, the timer will begin to time down and continue until it reaches zero, as shown in Figure 17.

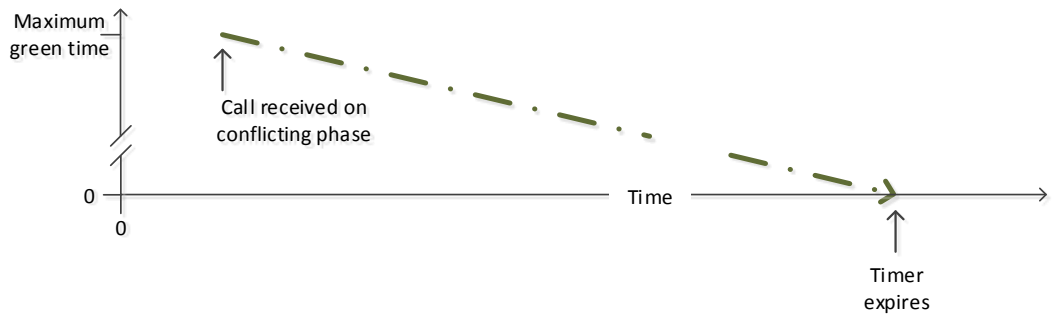


Figure 17. Maximum green timer process

Timing of a Phase

The phase termination logic in a controller determines how long a phase will time and when it will terminate. For standard actuated traffic control, a phase will continue to time until one of two possible events occur, either a gap out or a max out.

A *gap out* occurs when both the minimum green timer and the passage timer expire. An example of a gap out is shown in Figure 18. Even though the maximum green timer is still active and timing down, once the minimum green timer and the passage timer both expire, the phase will gap out.

A *max out* occurs when the maximum green timer expires. An example of a max out is shown in Figure 19. The minimum green timer expires, but continuing traffic demand extends the passage timer. It resets each time a new vehicle is detected. However, the phase terminates when the maximum green timer expires.

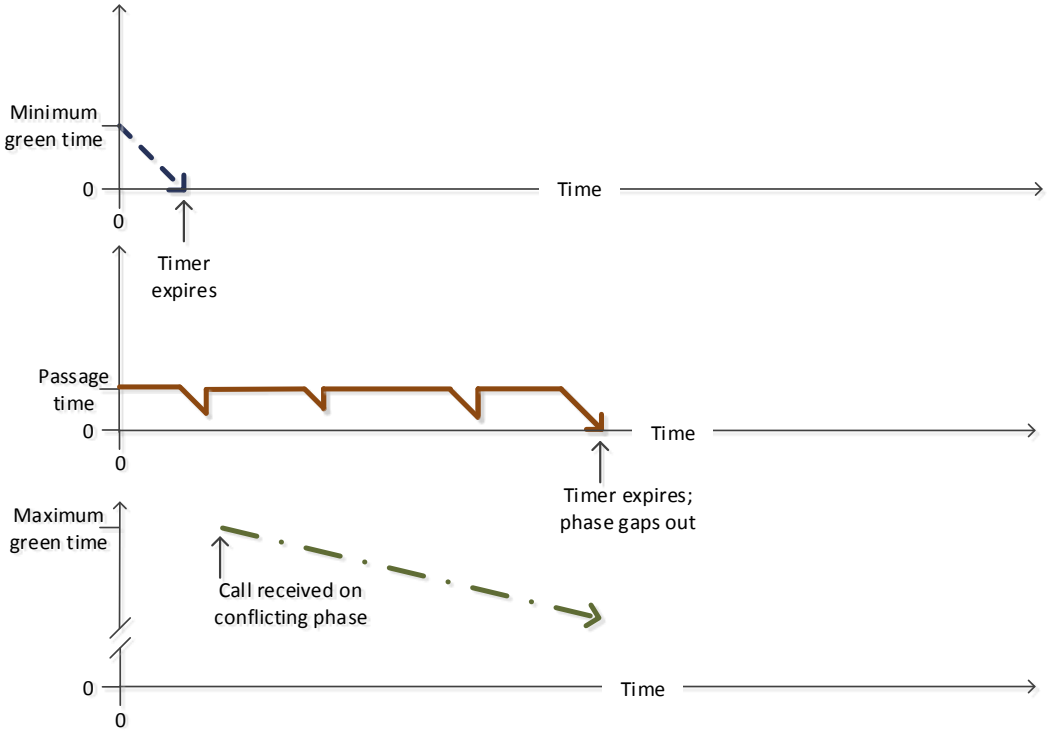


Figure 18. Example of gap out

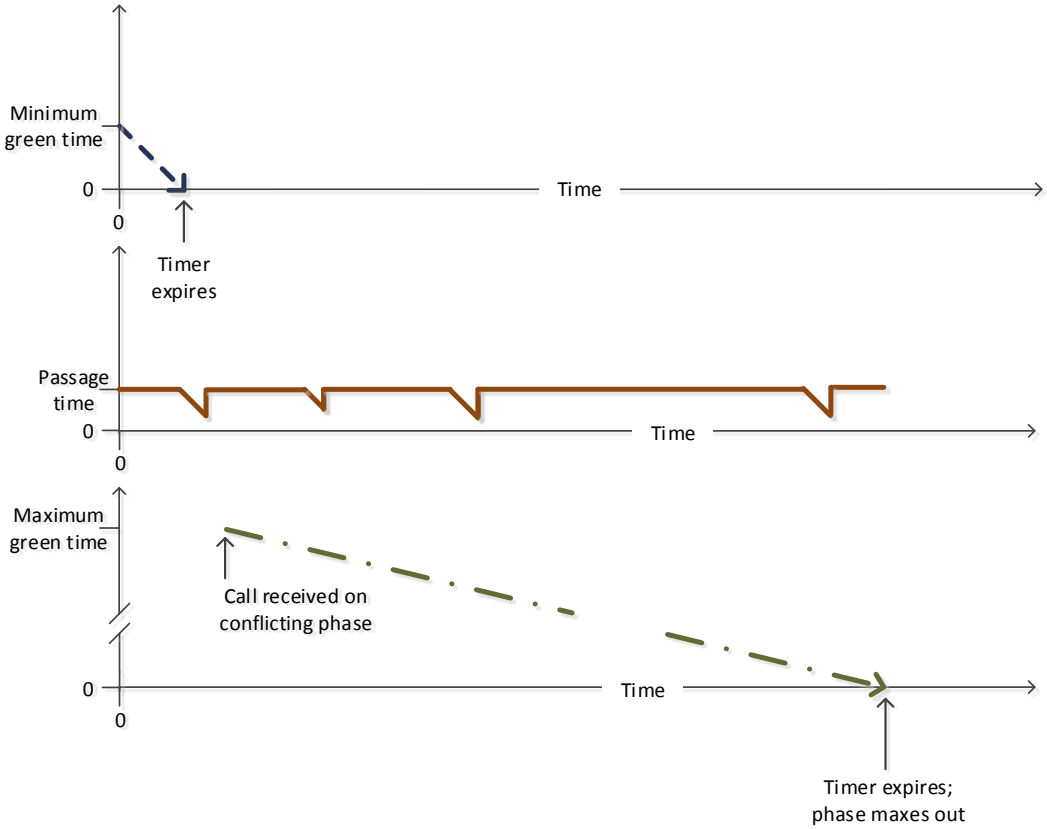


Figure 19. Example of max out

Example Calculation 4. Determining Timing Processes and Phase Termination

Figure 20 shows a traffic control process diagram with vehicle trajectories in a time-space diagram format. The front of the vehicle trajectory is shown with the solid line while the rear of the vehicle is shown with the dashed line. Six vehicles are in queue at the beginning of green, when t = 0 sec. Based on a minimum greet time of 3 sec, a passage time of 3 sec, and a maximum green time of 20 sec, draw the detector status, the timer status, and the display status. Show the graphs for the values of the three timing processes, noting the maximum and minimum values for each of the processes on the y-axis. The resulting signal display may change some of the vehicle trajectory plots. Note on the figure where you think that these changes will occur. Assume a yellow time of 3 sec. Assume that the conflicting call is first received at t = 0 and continues throughout the duration of green.

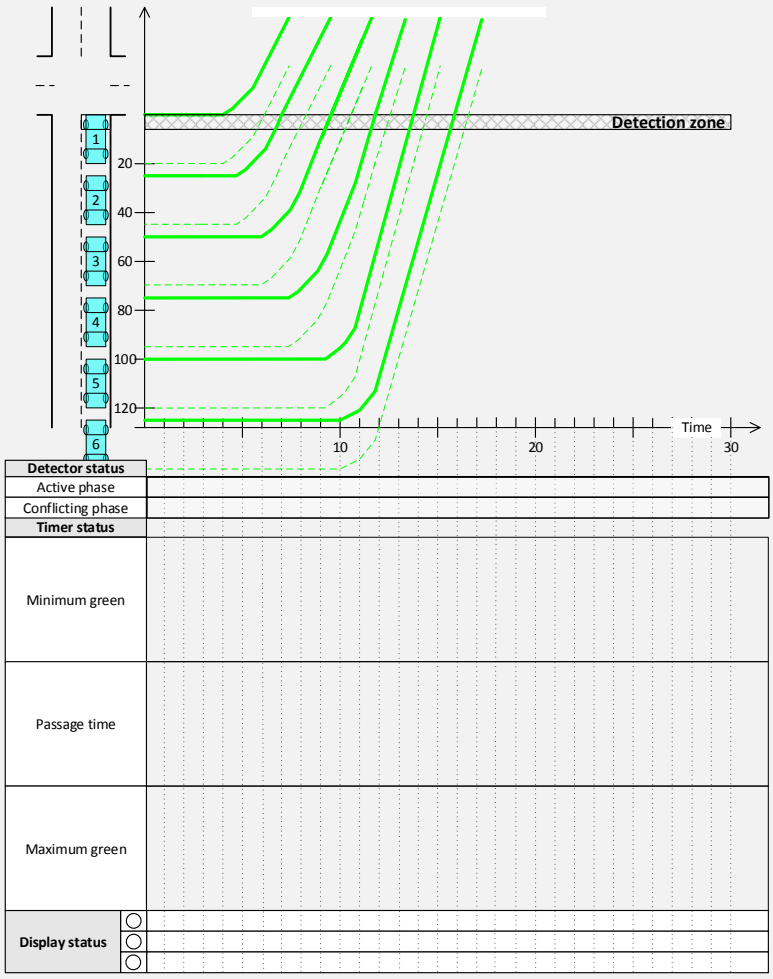


Figure 20. Data and traffic control process diagram for Example Calculation 4

Figure 21 shows the result with the timing processes and the signal display status, based on the traffic flow (as represented by the vehicle trajectories) and the signal control (as represented by the timing parameters). The passage timer expires just before the maximum green timer expires. Thus the phase gaps out. Note that all six vehicles are able to travel through the intersection as original shown in the time-space diagram.

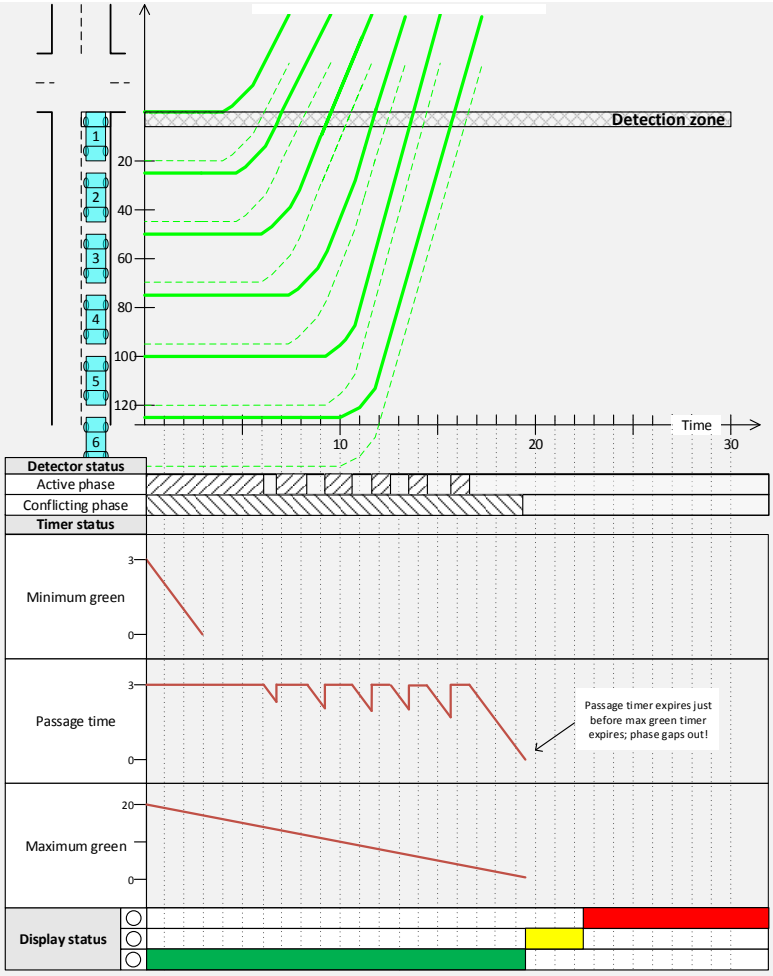


Figure 21. Solution for Example Calculation 4

Example Calculation 5. Determining Timing Processes and Phase Termination

Figure 22 shows a traffic control process diagram with the status of the active and conflicting phasing, as well as the timing parameter values. Based on a minimum green time of 0 sec, a passage time of 2 sec, and a maximum green time of 15 sec, show the resulting timing processes until the phase terminates. Also show the resulting signal display status. State how the phase terminates. Assume a yellow time of 3 sec and that the green starts at t = 0 sec.

Figure 23 shows the result with the timing processes and the signal display status, based on the detector status and the signal control (as represented by the timing parameters). The passage timer expires after one extension, and since the minimum green timer has expired, the phase gaps out.

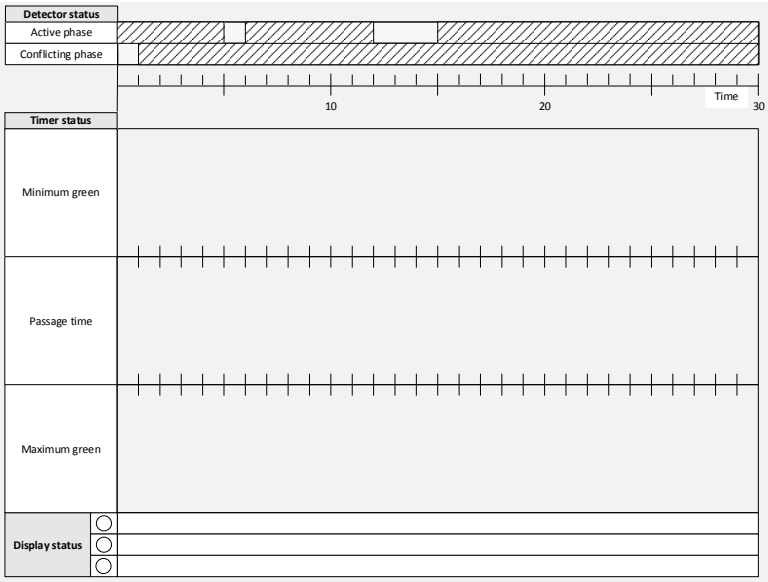


Figure 22. Data and traffic control process diagram for Example Calculation 5

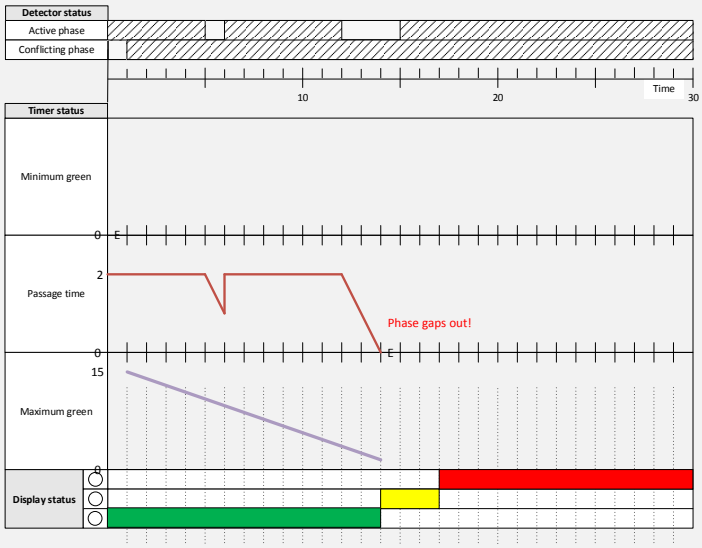


Figure 23. Solution for Example Calculation 5

5. Formulating the Model

We use a queuing model to represent traffic flow conditions on one lane or approach of a signalized intersection based on the conditions that we've observed in the field. This queuing model consists of the following parts:

- Vehicles arrive at the intersection in a uniform manner, with equal headways between vehicles.
- Vehicles depart from the intersection at three different rates, depending on the point in the cycle. During red, no vehicles depart. During the initial part of green while the queue is clearing, vehicles depart at the saturation flow rate. After the queue clears, vehicles depart at the same rate at which they arrive.

These arrival and departure processes are represented by three related diagrams, the flow profile diagram, the cumulative vehicle diagram, and the queue accumulation polygon. Before we present these diagrams, we illustrate the vehicle flow at a signalized intersection using a time-space diagram.

Vehicle Trajectories at a Signalized Intersection

The flow of vehicles approaching and traveling through a signalized intersection can be represented by a time-space diagram. A time-space diagram shows the position of each vehicle at any point in time, and the slope of its trajectory shows the speed of the vehicle at any point in time. The vehicular signal display intervals of green and red are also shown. The time that it takes to cycle through the display of these intervals is called the *cycle length*.

The time-space diagram in Figure 24 shows the flow of individual vehicles traveling through a signalized intersection (diagonal lines from lower left to upper right in the figure) during two complete signal cycles, illustrating several important concepts.

- During the first cycle, three vehicles arrive at the intersection during the green indication and travel through the intersection without stopping. The *arrival headway* h_a (the headway between vehicles arriving at a signalized intersection) is constant, a flow pattern called *uniform flow*. We assume the condition of uniform arrival flow in the queuing model considered here.
- During the second cycle, three vehicles arrive at the intersection during the red indication. A fourth vehicle arrives during the green indication but must initially stop because the vehicle in front of it is stopped. As each of these four vehicles arrives, a queue (waiting line) forms at the stop line. The trajectory of the four vehicles is horizontal during red, illustrating that while time passes, the vehicles are stationary. The position of each vehicle in the queue, and the spacing between vehicles in the queue, is shown in the time-space diagram.
- At the beginning of the green indication during cycle 2, the vehicles enter the intersection. The headway between vehicles in the departing queue is called the *saturation headway* h_s.
- A final vehicle arrives during cycle 2 after the queue has cleared and travels through the intersection without stopping.

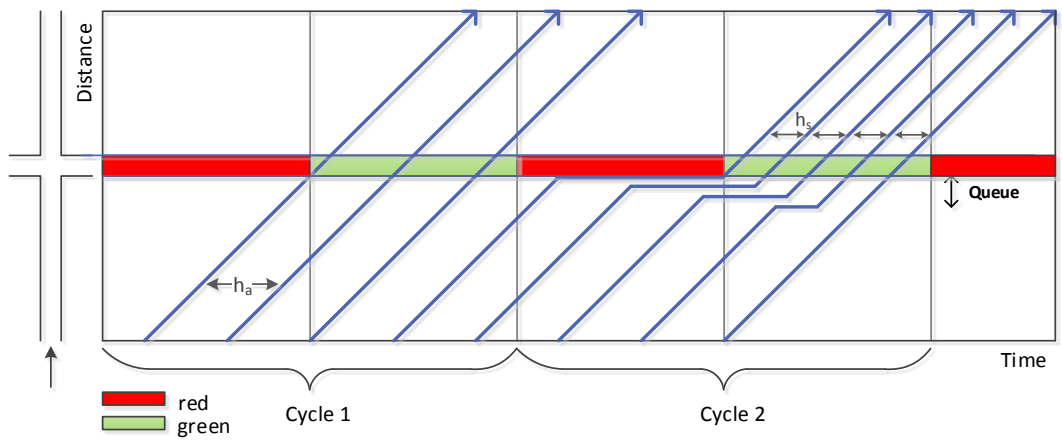


Figure 24. Vehicle trajectories (blue lines) represented in a time-space diagram

The Queuing Process at a Signalized Intersection

Traffic flow at a signalized intersection can be represented by the *D/D/1 queuing model*. The D/D/1 model assumes a deterministic arrival pattern, a deterministic service pattern, and one service channel. A deterministic arrival or service pattern means that the pattern is known and does not vary over time. Stated another way, there is no randomness in either pattern. One service channel implies one lane of an intersection approach. The model also assumes that the demand is less than the capacity.

Figure 25 shows these elements as applied to one lane on an approach to a signalized intersection. The *server* is the first vehicle position at the stop bar. This is the point at which vehicles are served as they exit the queuing system and enter the intersection. The *queue* forms behind the vehicle in the server and extends to some maximum point, depending on the arrival and service rates of the system. While queuing theory assumes that this line of vehicles is a vertical stack, in reality the queue extends horizontally upstream from the stop bar to the point of the maximum queue. This point is considered to be the entry point to the queuing system.

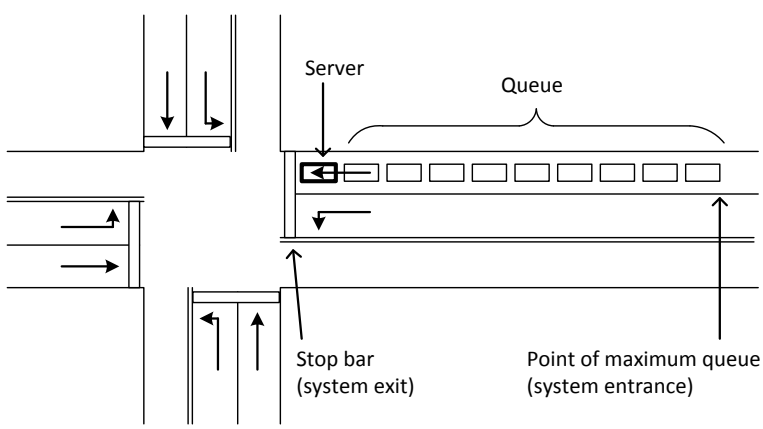


Figure 25. Elements of the queuing system

Figure 26 shows another view of the queuing process, here overlaid vertically on a time-space diagram (at the left side of the figure). The arrival pattern is shown just upstream of the signalized intersection, showing the constant headway between each of the vehicles. As noted above, this pattern is called *uniform arrivals* and is consistent with the deterministic arrival pattern for the

D/D/1 queuing model. The service or departure pattern is shown just downstream of the intersection for three time periods.

- During period 1, vehicles 1, 2, and 3 arrive during the red interval and form a queue. The service or departure rate is zero because the signal indication is red.
- When the signal display changes to green, vehicles 1, 2, and 3 depart from the intersection at the saturation flow rate. Here the headway between vehicles is equal to the saturation headway. A fourth vehicle arrives during green but joins the queue and can't be served until the queue clears. The service rate for this vehicle is also equal to the saturation flow rate. The time that it takes for the queue to clear (called the *queue service time* g_s) is the duration of the second period.
- During the third period, vehicles 5 and 6 arrive and depart at a constant rate, equal to the arrival rate. Since the queue has cleared, these vehicles experience no delay.

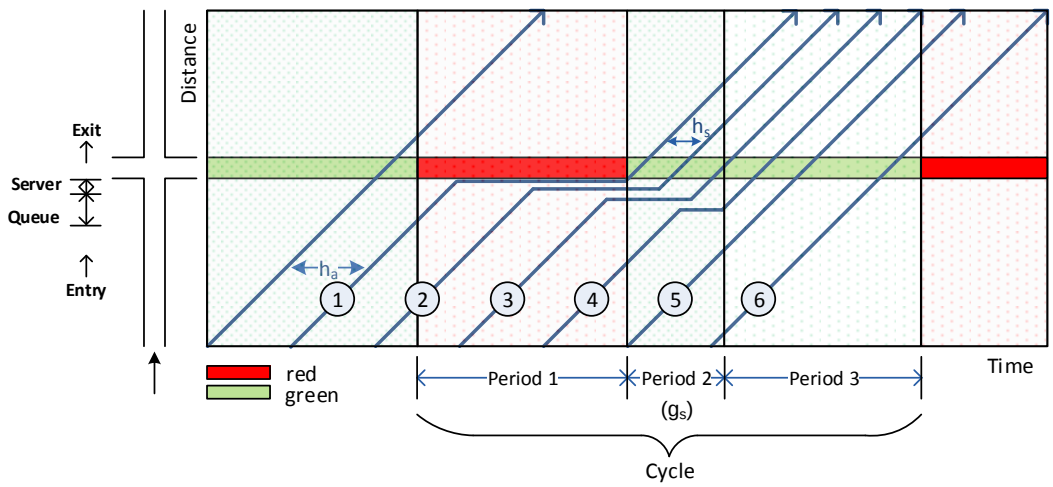


Figure 26. Trajectories representing vehicles arriving at and departing from a signalized intersection

There are three other ways to represent the queuing process including the flow profile diagram, the cumulative vehicle diagram, and the queue accumulation polygon. These diagrams also show important concepts in the traffic flow process such as capacity and delay, as well as other ways to represent intersection operation and performance, and are discussed below.

The Flow Profile Diagram

The *flow profile diagram* (Figure 27) represents both the arrival flow to and the departure flow from a signalized intersection over time. The flow profiles can be extracted from the time-space diagram shown in Figure 26 either by calculating the flow rate from the headway between vehicles, or by counting the number of vehicles over constant time segments. The arrival flow rate is constant (uniform) and is represented by the solid line in Figure 27. The service flow rate (represented by the dashed line) varies during the cycle, according to the three time periods noted in the discussion of the time-space diagram above. The service flow rate is equal to:

- Zero, during the red indication.
- The saturation flow rate *s*, while the queue is clearing, during an interval called the queue service time g_s.
- The arrival flow rate *v*, after the queue clears and until the end of the green interval.

Since the queue clears before the end of green, the volume is less than the capacity. This meets one of the assumptions of the D/D/1 queuing model established earlier. This service pattern is repeated for each signal cycle.

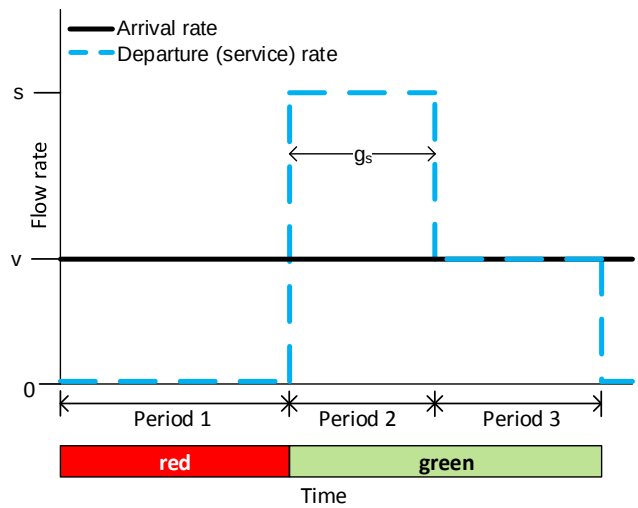


Figure 27. Flow profile diagram

Figure 28 shows that the server can be represented by a pipe with the arrival (input) flow profile diagram shown on the left and the service (output) flow profile shown on the right. The area of the pipe equals the saturation flow rate.

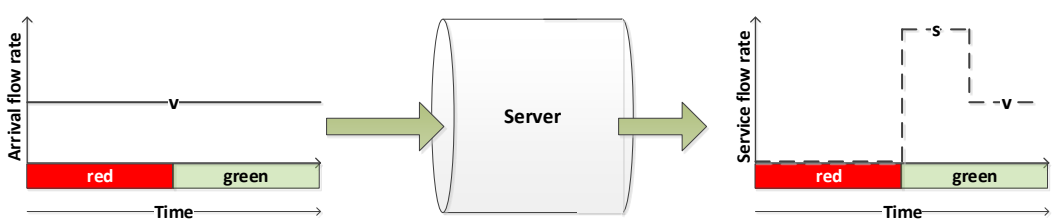


Figure 28. Queuing process showing input, server, and output

Example Calculation 6. Determining the Flow Profile Diagram

Prepare a flow profile diagram that represents the following conditions:

- Arrival flow rate = 600 veh/hr
- Saturation flow rate = 1900 veh/hr
- Queue service time = 13.8 sec
- Cycle length = 60 sec
- Green time = 30 sec
- Red time = 30 sec

The arrival flow profile is represented by a horizontal line whose value is a constant 600 veh/hr. The departure or service flow profile is represented by three line segments, one for each of the three periods described above:

- During red, the departure flow rate is zero.

- During the period that the queue is clearing, or the queue service time, the departure flow rate is equal to the saturation flow rate, or 1900 veh/hr.
- After the queue has cleared, the departure rate is equal to the arrival rate, or 600 veh/hr.

Figure 29 shows both the arrival and departure flow profiles for the conditions just described.

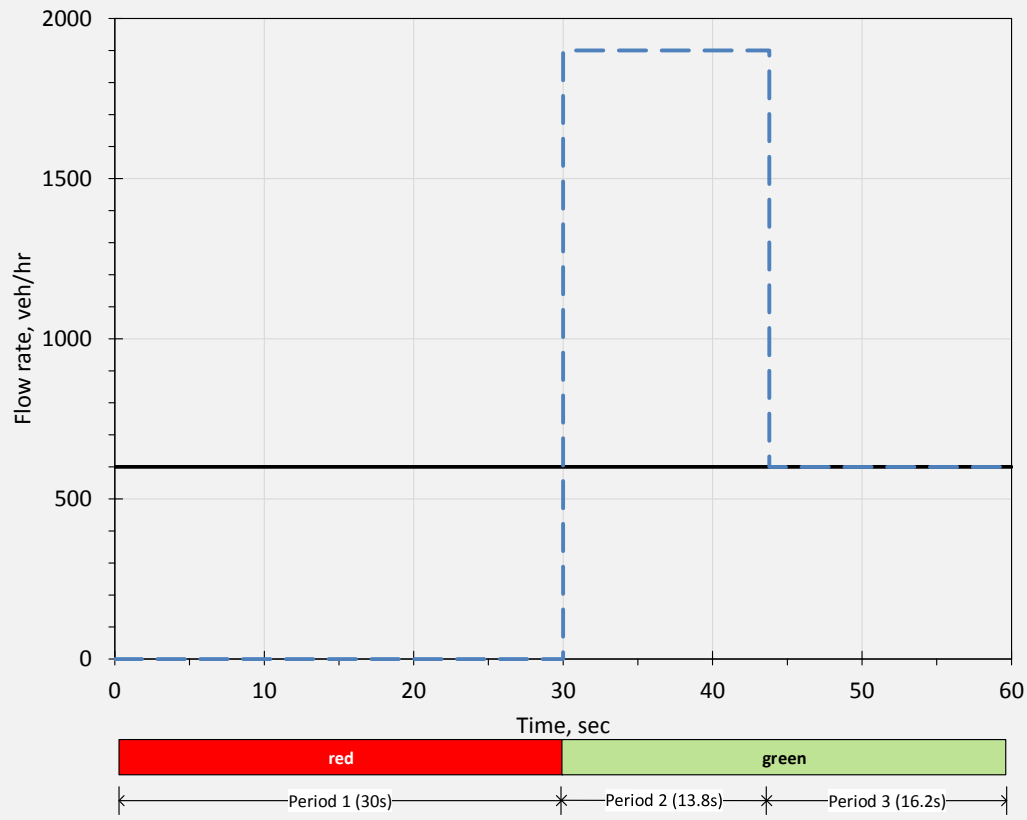


Figure 29. Example flow profile diagram for Example Calculation 6

The Cumulative Vehicle Diagram

The *cumulative vehicle diagram* (Figure 30) is a running total of the number of vehicles that have arrived at and departed from the intersection over time. The cumulative vehicle diagram shows two lines, one representing the cumulative number of arrivals over time and the other the cumulative number of departures.

When we assume that the arrival pattern is uniform, with constant headways, the horizontal line from the flow profile diagram becomes a line of constant slope in the cumulative vehicle diagram. The slope of the line representing vehicle arrivals is equal to the arrival flow rate. As we move from left to right in the diagram (representing the passage of time), the y-axis value shows the running total of the number of vehicles that have arrived at the intersection at any point in time.

The service pattern is again divided into three periods. During the red indication, no vehicles can depart from the intersection, so the total number of departures is zero during this period. Once the green is displayed, vehicles begin to depart from the intersection at the saturation flow rate, with headways between vehicles equal to the saturation headway. The slope of the departure line during this time interval is equal to the saturation flow rate. At the point that the queue clears, the

arrival line and the departure line become coincident. Their slopes are the same and equal to the vehicle arrival rate during the remainder of the green interval.

The cumulative vehicle diagram also shows three measures of intersection performance, indicating how well the intersection is operating.

- The first measure is the length of the queue at any point in time. The *length of the queue* is the difference between the number of vehicles that have arrived at and departed from the intersection at any point in time. Graphically, the queue length (or number of vehicles currently in the system, Q) is the vertical distance between the arrival line and the departure line, as illustrated in Figure 30.

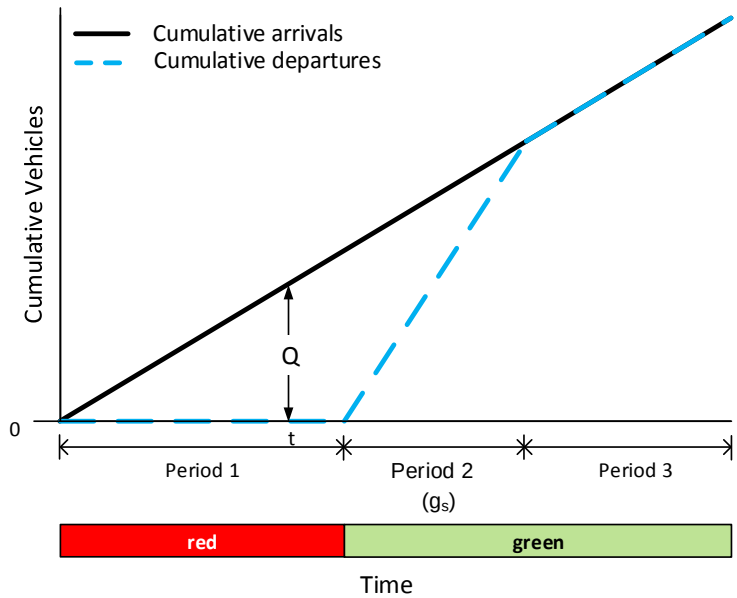


Figure 30. Cumulative vehicle diagram showing queue length at time t

- The second measure is the time that each vehicle spends in the system, or the *delay* that it experiences. Consider the horizontal line connecting the arrival line and the departure line for the vehicle (noted as vehicle i) shown in Figure 31. Point 1 on the arrival line is the time that the vehicle enters the system; point 2 on the departure line is the time that the vehicle exits the system. The time interval between these two points is the delay experienced by the vehicle.
- The third measure is the *total delay* experienced by all vehicles that arrive at and travel through the intersection. If we add all of the horizontal lines described in the bullet above for all vehicles, we get the total delay experienced by these vehicles. The total delay is the area of the triangle formed by the arrival and departure lines and shown as the shaded area in Figure 32.

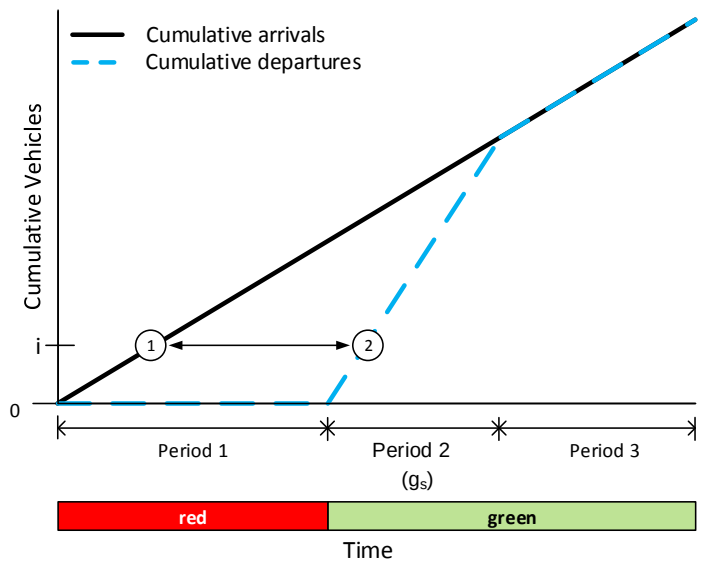


Figure 31. Cumulative vehicle diagram showing delay for vehicle i

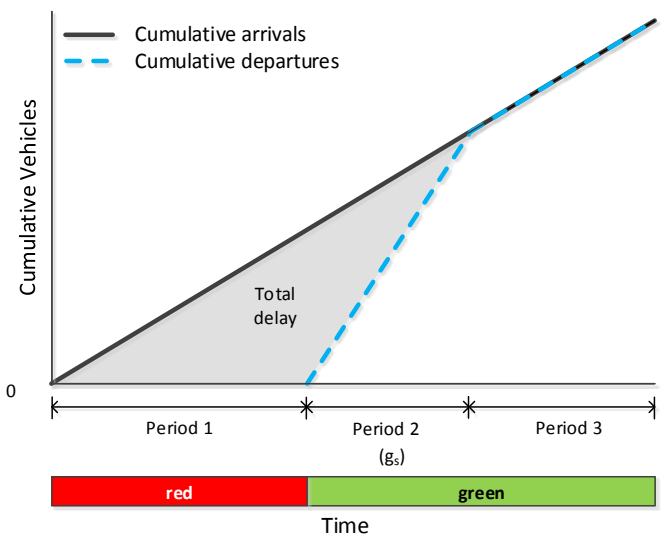


Figure 32. Cumulative vehicle diagram showing total delay

Example Calculation 7. Determining the Cumulative Vehicle Diagram

Field data collected on one approach of a signalized intersection showed that:

- Vehicles arrive every 6 sec at a uniform rate.
- The cycle length is 60 sec, with red and green time intervals of 30 sec each.
- Vehicles depart every 2 sec after the beginning of green.
- The queue service time is 14 sec.

Prepare a cumulative vehicle diagram that represents these conditions.

The cumulative vehicle diagram for these conditions is shown in Figure 33. Since a vehicle arrives at the intersection every six seconds, a cumulative total of five vehicles arrive from the

beginning of red until the end of red. We've assumed that the vehicle that arrives at t = 0 (end of green) travels through the intersection without stopping. Also, since we are dealing with the discrete events of vehicle arrivals in the field, the lines are "stair step" with each increase representing a vehicle arrival or departure. By contrast, the theoretical depictions presented earlier are based on a continuous and not discrete process.

The chart shows that at t = 44 sec, the cumulative number of vehicles that have entered the system (seven) equals the number that have exited. It is at this point (14 sec after the start of the green interval) that the arrival and departure lines become coincident and the queue clears.

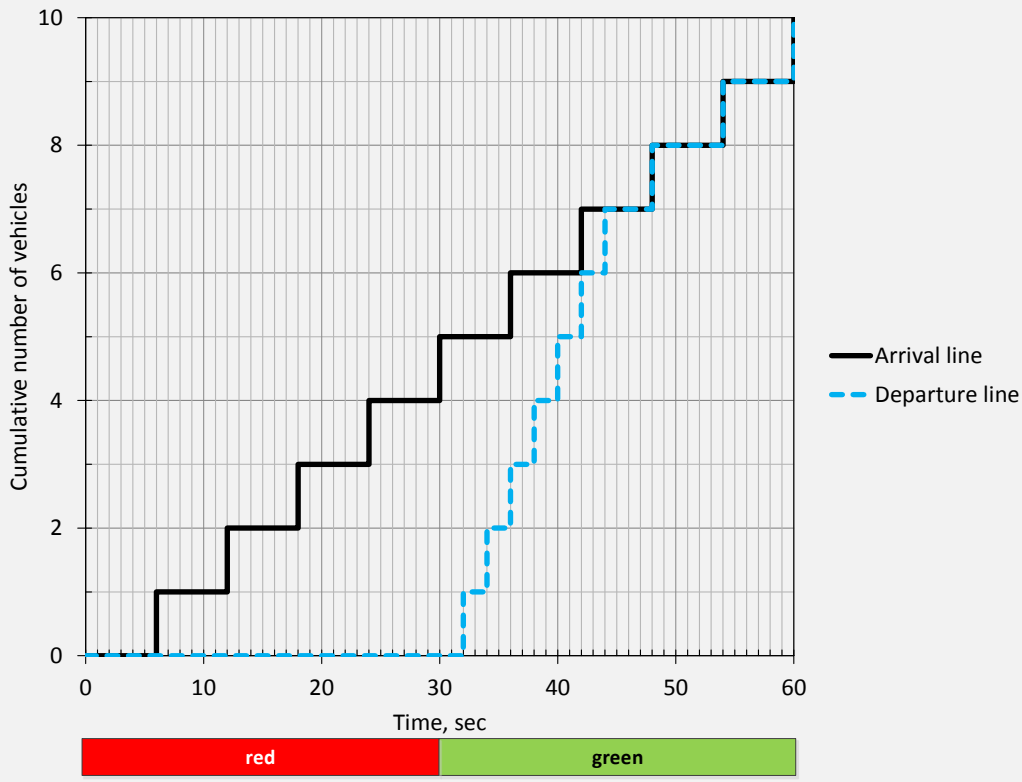


Figure 33. Example cumulative vehicle diagram

What is the delay for the vehicles that arrive at the intersection? Table 5 shows the times that each vehicle arrives at and departs from the intersection, as read from the cumulative vehicle diagram (Figure 33) for the first seven vehicles. The difference between the arrival and departure times is the time in the system or the delay experienced by each vehicle.

Table 5. Vehicle arrivals and departures (and time in system)

Vehicle number	Arrival time (sec)	Departure time (sec)	Delay (time in system), (sec/veh)
1	6	32	26
2	12	34	22
3	18	36	18
4	24	38	14
5	30	40	10
6	36	42	6
7	42	42	2
		Total	98 sec

Vehicle #1 has the longest delay (26 sec) since it arrives near the beginning of the red interval. The seventh vehicle arrives just as the queue is clearing and has a delay of about 2 sec. Vehicles 8, 9, and 10 arrive and leave without delay, as the queue has cleared by the time that vehicle 8 arrives. The total delay for all vehicles is 98 sec.

The Queue Accumulation Polygon

The *queue accumulation polygon* represents the length of the queue at any point in time and is derived from the cumulative vehicle diagram. Figure 34 shows the queue accumulation polygon, again for the case of uniform arrivals: the queue grows during red and reaches its maximum length at the end of the red interval (or the beginning of the green interval). It decreases once the green interval begins and reaches zero when the queue clears. It remains at zero until the end of the green interval.

The area of the queue accumulation polygon is equal to the area between the arrival and departure lines of the cumulative vehicle diagram: both areas represent the total delay experienced by all vehicles that arrive and leave during the cycle.

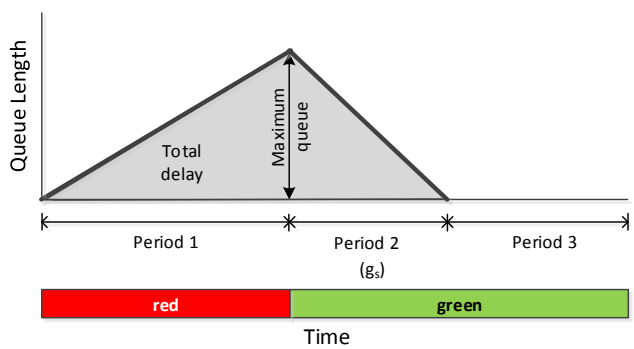


Figure 34. Queue accumulation polygon

Example Calculation 8. Determining the Queue Accumulation Polygon

Consider the conditions given for the cumulative vehicle diagram in Example Calculation 2. Prepare a queue accumulation polygon that represents these conditions.

The queue accumulation polygon represents the length of the queue over time and is shown in Figure 35 for the given conditions. The queue accumulation polygon shows the queue growing during red and reaching a maximum length of 5 vehicles at the end of red (t = 30 sec). The queue begins to clear when the display changes from red to green and clears 14 sec later (t = 44 sec). The queue remains at zero after t = 44 sec, as vehicles arrive and depart without delay. Figure 35

represents the discrete form of the queue accumulation polygon where the change for each vehicle is shown, in contrast to the continuous form shown in Figure 34.

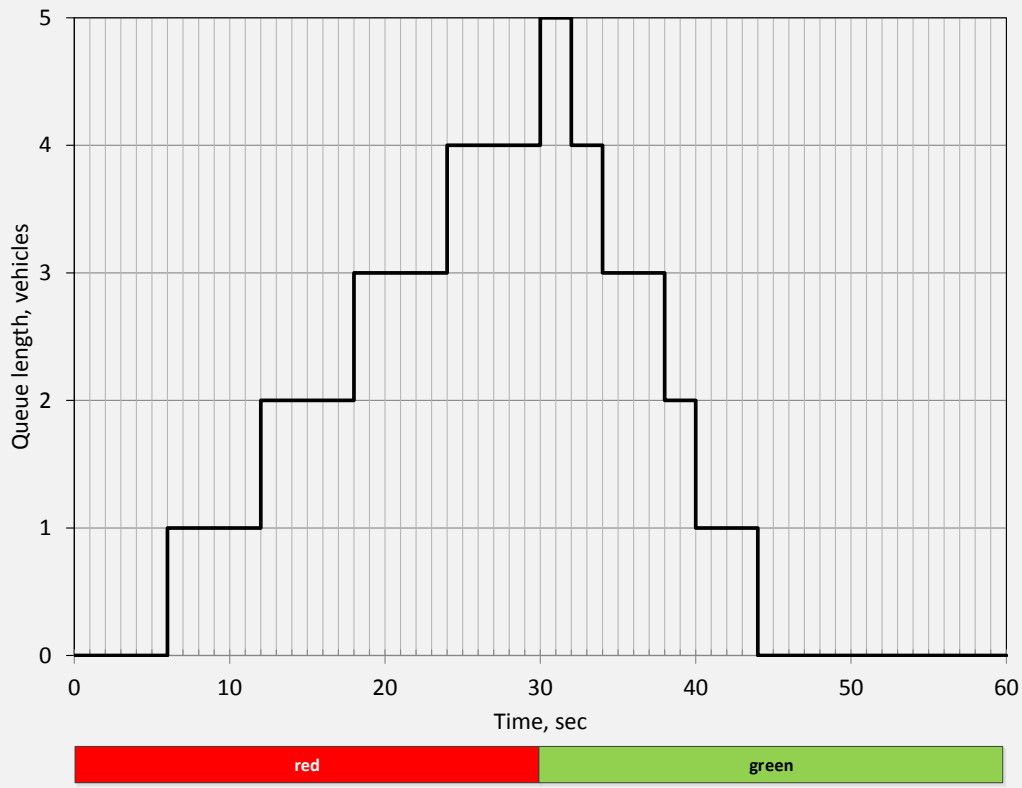


Figure 35. Queue accumulation polygon for Example Calculation 8

In summary, we have represented traffic flow on one lane at a signalized intersection as a queuing process. The arrival pattern consists of uniform flow with a constant rate. The service pattern is represented by three values: zero during red, the saturation flow rate during the queue clearance process, and the arrival flow rate after the queue has cleared. The process can be represented by a flow profile diagram, a cumulative vehicle diagram, and a queue accumulation diagram. Each of these diagrams will be used in the following sections of this chapter.

6. Scenario 1. Calculating the Capacity of an Intersection Approach

The capacity of an approach to a signalized intersection depends on two factors: the maximum flow rate that can be achieved by vehicles departing from the intersection and the proportion of the cycle that is available to serve this approach. To illustrate the calculation of the approach capacity, we will use Scenario 1, an intersection of two one-way one-lane streets, with through movements only, and controlled by a pretimed traffic signal. We will show how to calculate the capacity of movement 2, as illustrated in Figure 36.

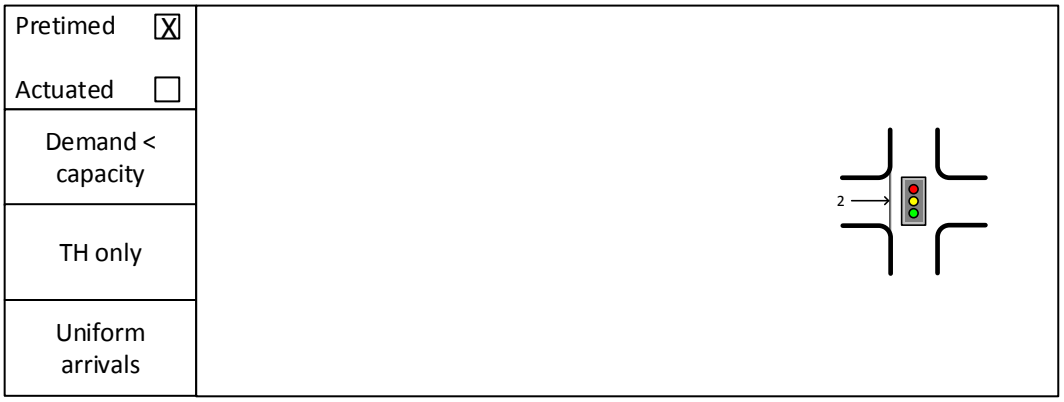


Figure 36. Scenario 1

Effective Green Time and Effective Red Time

The green time available to serve traffic is called the *effective green time*. The effective green time for a given movement is the sum of the displayed green, yellow, and red clearance interval times minus the total lost time. Lost time is the time during the cycle unavailable to vehicles either from the starting of the queue at the beginning of green or the transition to red at the end of green.

Equation 1

$$g = G + Y + RC - t_L$$

where

g = effective green time, sec,
G = displayed green time, sec,
Y = yellow time, sec,
RC = red clearance time, sec, and
t_L = total lost time for a phase, sec.

The *effective red time* is the time during the cycle that is not available to serve traffic. The effective red time r is calculated as

Equation 2

$$r = C - g$$

where

C = cycle length, sec, and
g = effective green time, sec.

The relationship between the displayed green time *G*, the effective green time *g*, and the startup (l_1) and clearance (l_2) lost times is represented graphically in Figure 37. Note that e is the extension of effective green and is assumed to be 2.0 sec.

Figure 37. Relationship between displayed green time, effective green time, and lost time

Saturation Flow Rate, Green Ratio, and Capacity

While the *saturation flow rate* is the maximum number of vehicles that can pass by the stop line on a lane or approach if the green indication is displayed continuously for an hour, the *capacity* is the maximum flow rate that would be observed based on the amount of green time that is actually available.

The proportion of the hour that is effectively available for a given movement is called the *effective green ratio*, or the ratio of the effective green time to the cycle length. The capacity of an approach or a lane is thus defined as the product of the saturation flow rate and the effective green ratio, as given in Equation 3.

Equation 3

$$c = s\left(\frac{g}{C}\right)$$

where

c = capacity of an approach or lane, veh/hr,
s = saturation flow rate, veh/hr, and
g/C = effective green ratio (effective green time divided by cycle length).

Example Calculation 9. Determining Approach Capacity

For one approach of a signalized intersection (Figure 38), the saturation flow rate is 1900 vehicles per hour. The green is displayed for 15 sec, while the sum of the yellow and red clearance displays is 5 sec. The lost time is 4 sec. There are 60 cycles in one hour. What is the capacity of the approach?

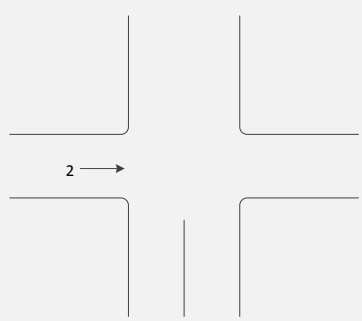


Figure 38. Intersection for Example Calculation 9

Since there are 60 cycles/hr, we know that the cycle length is:

$$C = \frac{(3600\ sec/hr)}{(60\ cycle/hr)} = 60\ sec$$

The effective green time is computed using Equation 1:

$$g = G + Y + R_c - t_L = 15\ sec + 5\ sec - 4\ sec = 16\ sec$$

The effective green ratio is given by

$$\frac{g}{C} = \frac{16\ sec}{60\ sec} = 0.27$$

The capacity of the approach is the product of the saturation flow rate and the effective green ratio, using Equation 3.

$$c = 1900\ veh/hr \times 0.27 = 513\ veh/hr$$

7. Scenario 2. Calculating the Capacity Utilization of an Intersection Using Critical Movement Analysis

When designing a new intersection or evaluating the operation of an existing intersection, a common question to ask is: what is the capacity and is it sufficient to accommodate the traffic volume? A measure often used to determine whether there is sufficient capacity on an intersection approach is the volume-to-capacity ratio. A common method used to determine the volume-to-capacity ratio for an entire intersection is the critical movement analysis. Scenario 2, shown in Figure 39, is used to illustrate the determination of capacity utilization using the critical movement analysis method. Scenario 2 is based on an intersection with four approaches. Each approach has two lanes, an exclusive left turn lane and a through lane. The intersection is controlled by a pretimed signal.

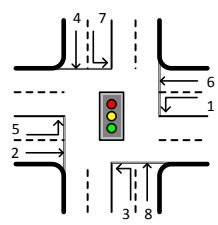
Pretimed ☒ Actuated ☐	
Demand < capacity	
Protected or permitted LTs	
Uniform arrivals	

Figure 39. Scenario 2

Flow Ratio and Volume-to-Capacity Ratio

The *flow ratio* is defined as the ratio of the volume for a given movement to the saturation flow rate for that movement, or

Equation 4

$$Y = v/s$$

where

Y = flow ratio,
v = volume, veh/hr, and
s = saturation flow rate, veh/hr.

The flow ratio is the proportion of an hour that is required to serve a traffic movement. Thus the flow ratio determines the minimum effective green ratio required to serve that movement.

Example Calculation 10. Calculating Flow Ratio and Effective Green Ratio

Suppose that the volume on one approach of a signalized intersection is 600 veh/hr and the saturation flow rate for the approach is 1900 veh/hr. What proportion of the hour should be made available to serve this movement so that sufficient capacity is provided?

Using Equation 4, the flow ratio Y is determined to be

$$Y = \frac{600\ veh/hr}{1900\ veh/hr} = 0.32$$

This means that the effective green ratio must be at least 0.32 if sufficient capacity is to be provided to serve the demand on this approach.

The *volume-to-capacity ratio* is defined in Equation 5.

Equation 5

$$X = v/c$$

where
X = volume-to-capacity ratio,
v = volume, veh/hr, and
c = capacity, veh/hr.

Since the capacity is the product of the saturation flow rate and the effective green ratio, we can rewrite the definition of the volume-to-capacity ratio as

Equation 6

$$X = \frac{v}{s\left(\frac{g}{C}\right)} = \frac{v/s}{g/C} = \frac{Y}{g/C}$$

where
v = volume, veh/hr,
s = saturation flow rate, veh/hr,
g = effective green time, sec, and
C = cycle length, sec.

We can see from Equation 6 that if the flow ratio is less than the effective green ratio, then the volume-to-capacity ratio will be less than one. In this case, there will be sufficient capacity to serve the traffic demand. However, if the flow ratio is greater than the effective green ratio, there will not be sufficient capacity to serve the demand.

Example Calculation 11. Volume-to-Capacity Ratio

Suppose that the volume on an intersection approach is 750 veh/hr while the saturation flow rate is 1900 veh/hr. If the effective green ratio is 0.42, what is the volume-to-capacity ratio for the approach?

We first compute the capacity of the approach:

$$c = s\left(\frac{g}{C}\right) = 1900\ veh/hr\ (0.42) = 798\ veh/hr$$

The volume-to-capacity ratio is then:

$$X = \frac{v}{c} = \frac{750\ veh/hr}{798\ veh/hr} = .94$$

There are several interpretations of this result that are worth noting. First, there is just enough capacity to serve the demand, as *X* is just under one. However, if there is any variation in the demand (even small variations from one cycle to the next), the demand may exceed capacity. So, as we will see in the next section, it is prudent to provide some margin of safety in the design of the intersection and the signal timing. Often a design value used for the volume-to-capacity ratio is 0.85 or less.

Critical Movement Analysis

While the definitions for volume-to-capacity ratio and flow ratio apply to an individual movement, we often want to determine the volume-to-capacity ratio for the entire intersection. If we know the volume-to-capacity ratio for the entire intersection, we can answer the question that began this section: is there sufficient capacity at the intersection to accommodate the traffic volume existing at or projected for the intersection?

Critical movement analysis is the tool that we use to assess whether there is sufficient capacity available at the intersection to accommodate the demand. The outcome of a critical movement analysis is an estimate of the critical degree of saturation, X_c, and a determination if the intersection will operate below, near, or above capacity. The critical movement analysis is based on the following two points.

- For each movement, what proportion of the hour is needed in green time to serve the movement? To answer this question, we use the flow ratio: Y = v/s.
- For each concurrency group, we determine the sequence of conflicting movements that require greatest proportion of the hour to be served. This results in the concept of the critical sum, or the flow ratio sum that is the highest for each concurrency group. This requires that we know the phase sequencing for each ring, based on the ring barrier diagram.

The critical movement analysis includes five steps:

1. Compute the flow ratio for each movement.
2. Determine the flow ratio sums.
3. Within each concurrency group, identify movement pairs with the maximum flow ratio sum.
4. Determine the critical degree of saturation for the intersection.
5. Determine the sufficiency of capacity.

The method is described in detail in the following pages.

Step 1: Compute the flow ratio Y_i for each movement i present at the intersection. The standard movement numbers and notations are shown in Figure 40. Right turn volumes are combined with the through movement volumes.

Equation 7

$$Y_i = \frac{v_i}{s_i}$$

where

Y_i = flow ratio for movement i,

v_i = volume for movement i, veh/hr, and
s_i = saturation flow rate for movement i, veh/hr.

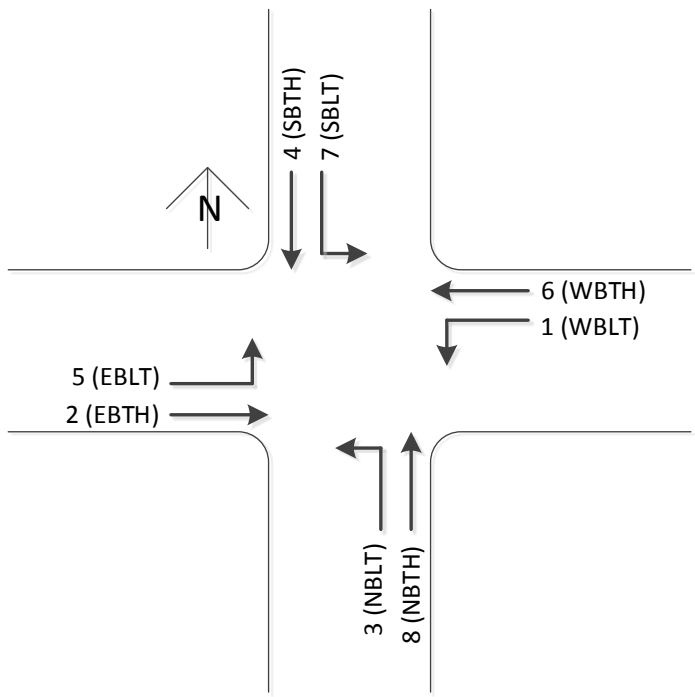


Figure 40. Numbering and notation of movements at a signalized intersection

Step 2: Determine the flow ratio sums for the phase sequences in each ring for each concurrency group (for the case of protected left turns only). (For permitted left turns, skip to step 3.)
Figure 41 and Figure 42 show the movements and phases from the east-west concurrency group. For the east-west concurrency group, the flow ratio sums for the movements served in rings 1 and 2 are given by Equation 8 and Equation 9.

Equation 8

$$Y_{EW1} = Y_1 + Y_2$$

Equation 9

$$Y_{EW2} = Y_5 + Y_6$$

where
Y_{EW1} = flow ratio sum for ring 1,
Y_{EW2} = flow ratio sum for ring 2, and
Y_i = flow ratios for movements 1, 2, 5, and 6

Figure 43 and Figure 44 show the movements and phases from the north-south concurrency group.

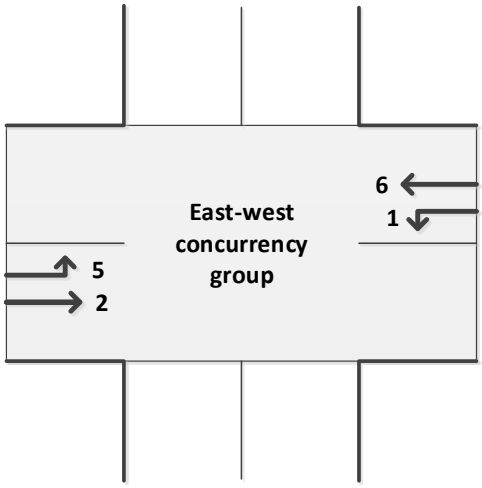


Figure 41. Movements served in east-west concurrency group

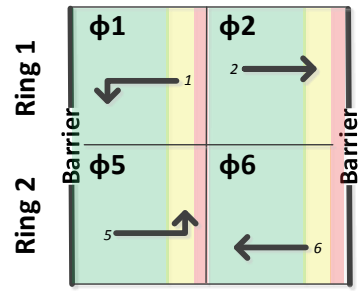


Figure 42. Phases served in east-west concurrency group

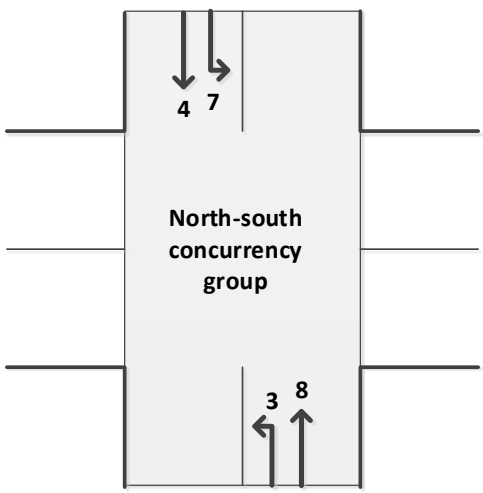


Figure 43. Movements served in north-south concurrency group

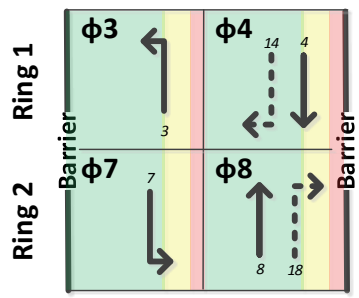


Figure 44. Phases served in north-south concurrency group

For the north-south concurrency group, the flow ratio sums are given by Equation 10 and Equation 11.

Equation 10

$$Y_{NS1} = Y_3 + Y_4$$

Equation 11

$$Y_{NS2} = Y_7 + Y_8$$

where

Y_{NS1} = flow ratio sum for ring 1,
Y_{NS2} = flow ratio sum for ring 2, and
Y_i = flow ratios for movements 3, 4, 7, and 8.

Step 3: Within each concurrency group, identify the movements with the maximum flow ratio sum (for protected left turns) or the movement with the maximum flow ratio (for permitted left turns).
These movements are the critical movements for each concurrency group.
For protected left turns, we use Equation 12 and Equation 13 to determine the critical movements for the east-west and north-south concurrency groups.

Equation 12

$$Y_{EW-critical} = Max(Y_{EW1}, Y_{EW2})$$

where

$Y_{EW\text{-}critical}$ = critical flow ratio for the EW concurrency group,
Y_{EW1} = flow ratio sum for ring 1, and
Y_{EW2} = flow ratio sum for ring 2.

Equation 13

$$Y_{NS-critical} = Max(Y_{NS1}, Y_{NS2})$$

where

$Y_{NS\text{-}critical}$ = critical flow ratio for NS concurrency group,
Y_{NS1} = flow ratio sum for ring 1, and
Y_{NS2} = flow ratio sum for ring 2.

For permitted left turns, we use Equation 14 and Equation 15 to determine the critical movements for each concurrency group.

Equation 14

$$Y_{EW-critical} = Max(Y_1, Y_2, Y_5, Y_6)$$

where

$Y_{EW\text{-}critical}$ = critical flow ratio for the EW concurrency group, and
Y_i = flow ratios for movements 1, 2, 5, and 6.

Equation 15

$$Y_{NS-critical} = Max(Y_3, Y_4, Y_7, Y_8)$$

where

$Y_{NS\text{-}critical}$ = critical flow ratio for NS concurrency group, and
Y_i = flow ratios for movements 3, 4, 7, and 8.

Step 4: Determine the critical volume-to-capacity ratio for the intersection.
Step 4a: Compute the lost time per cycle *L* as given in Equation 16.

Equation 16

$$L = \sum_{i=1}^{M} t_{Li}$$

where

L = lost time per cycle, sec,

t_{Li} = lost time for phase i, and
M = number of phases that serve the critical movements for one cycle.

If all left turn movements are protected, M is 4; if all are permitted, M is 2. If the left turn movements for one concurrency group are protected and if they are permitted for the other concurrency group, M is 3.

Step 4b: Compute the *critical volume-to-capacity* ratio considering the critical flow ratios and the lost time per cycle, using Equation 17.

Equation 17

$$X_c = \frac{(Y_{EW-critical} + Y_{NS-critical})(C)}{C - L}$$

where
X_c = critical volume-to-capacity ratio,
$Y_{EW\text{-}critical}$ = critical flow ratio for the EW concurrency group,
$Y_{NS\text{-}critical}$ = critical flow ratio for the NS concurrency group,
C = cycle length, sec, and
L = total lost time per cycle for the critical phases, sec.

Step 5: Based on the value of X_c calculated in step 4, determine the sufficiency of capacity. Table 6 gives four possible sufficiency ratings based on the critical volume-to-capacity ratio computed for the intersection. It is desirable, though not often possible during the peak period, for X_c to be less than 0.85.

Table 6. Sufficiency of capacity

X_c	Sufficiency of capacity rating
< 0.85	Intersection is operating under capacity. Excessive delays are not experienced.
0.85–0.95	Intersection is operating near its capacity. Higher delays may be expected, but continuously increasing queues should not occur.
0.95–1.00	Unstable flow results in a wide range of delay. Intersection improvements will be required soon to avoid excessive delays.
> 1.00	The demand exceeds the available capacity of the intersection. Excessive delays and queuing are anticipated.

Example Calculation 12

Critical Movement Analysis for Protected Left Turns

A standard 4-approach intersection has the geometric and volume characteristics shown in Figure 45. Figure 46 shows the phasing plan for this intersection, including leading protected left turns. The cycle length is 90 sec and the lost time is 4 sec/phase. The saturation flow rate is 1900 veh/hour/lane for through movements and protected left turn movements. Use the critical movement analysis method to determine whether the capacity is sufficient to serve the volume for this intersection.

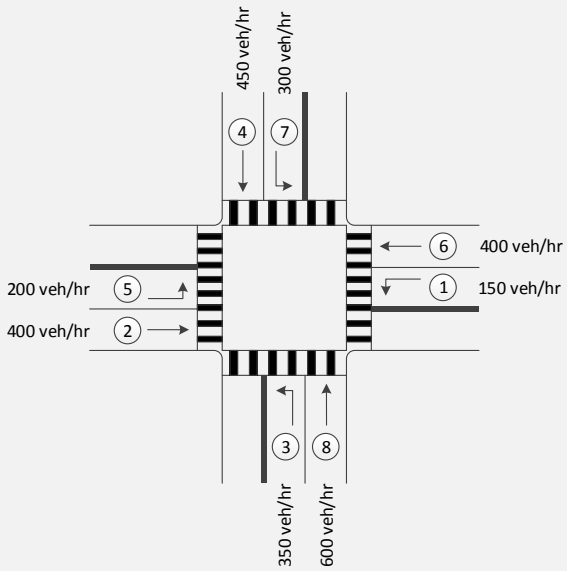


Figure 45. Intersection characteristics for Example Calculation 12

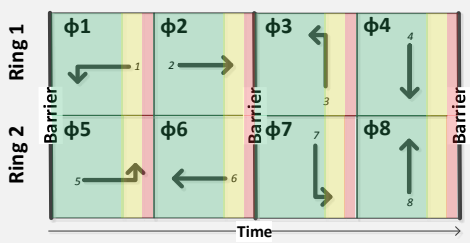


Figure 46. Phasing plan for Example Calculation 12

Step 1: Compute the flow ratio Y for each movement present at the intersection.
The flow ratios for each of the movements are calculated and shown in Figure 47.

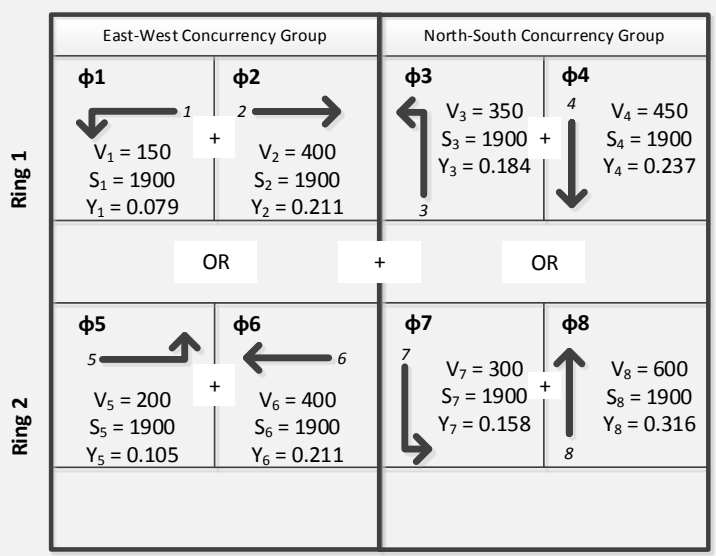


Figure 47. Flow ratios for each movement for Example Calculation 12

Step 2: Determine the flow ratio sums for the phase sequences in each ring for each concurrency group (for the case of protected left turns only).
The flow ratio sums for each ring within each concurrency group are shown in Figure 48. For example, for ring 1 in the east-west concurrency group, the flow ratios for movements 1 and 2 are .079 and .211, respectively. Their sum, noted as Y_{EW1}, is .290.

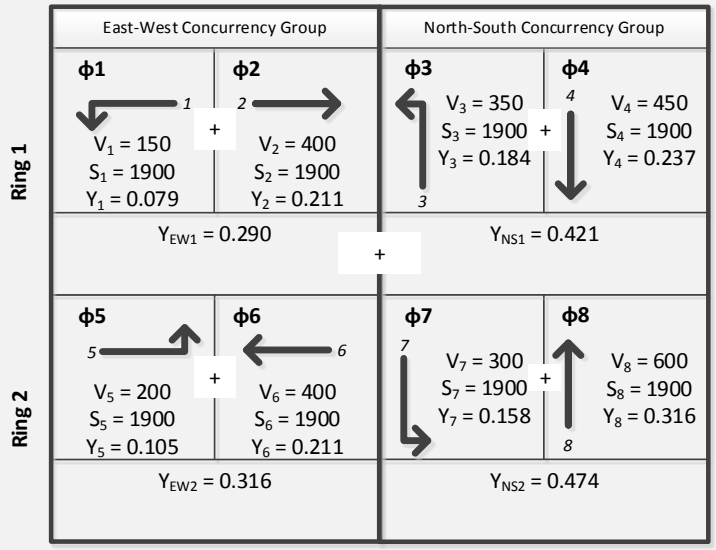


Figure 48. Flow ratio sums for Example Calculation 12

Step 3: Within each concurrency group, identify the movements with the maximum flow ratio sum (for protected left turns) or the movement with the maximum flow ratio (for permitted left turns). These movements are the critical movements for each concurrency group.
Since these are protected left turns, we identify the movements with the maximum flow ratio sum within each concurrency group (See Figure 49). For the east-west concurrency group, the movements served in ring 2 (movements 5 and 6) have the highest flow ratio sum (.316), as compared to the movements served in ring 1 (.290). For the north-south concurrency group, the movements served in ring 2 have the highest flow ratio sum (.474).

$$Y_{EW-critical} = Max(Y_{EW1}, Y_{EW2}) = Max(.290, .316) = .316$$
$$Y_{NS-critical} = Max(Y_{NS1}, Y_{SW2}) = Max\ (.421, .474) = .474$$

Step 4: Determine the critical volume-to-capacity ratio X_c for the intersection.
The critical flow ratios (Y_{EW2}=.316, Y_{NS2}=.474) were computed in step 3. The cycle length is given as 90 sec and the lost time per phase is 4 seconds. There are four critical phases since all left turns are protected, so the total lost time L is 16 sec. The critical volume-to-capacity ratio is computed using Equation 17:

$$X_c = \frac{(Y_{EW-critical} + Y_{NS-critical})(C)}{C - L} = \frac{(.316 + .474)(90\ sec)}{90\ sec - 16\ sec} = .96$$

Step 5: Based on the value of X_c calculated in step 4, determine the sufficiency of capacity.
The critical volume-to-capacity ratio, X_c = .96, indicates that the intersection is operating in the region of unstable flow and that excessive delays and queuing will result, using the ratings from Table 6.

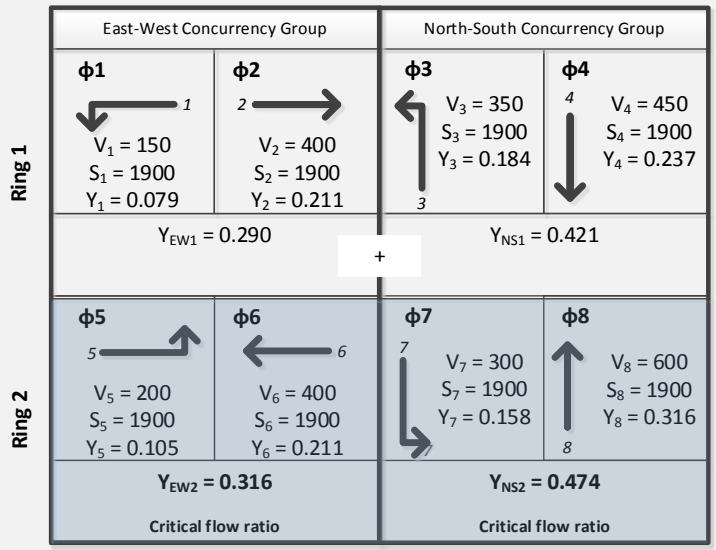


Figure 49. Critical flow ratios for Example Calculation 12

Example Calculation 13. Critical Movement Analysis for Permitted Left Turns

A standard four-approach intersection has the geometric and volume characteristics shown in Figure 50. The phasing scheme is shown in Figure 51. The cycle length is 90 sec and the lost time is 4 sec per phase. The permitted left turns have a saturation flow rate of 450 veh/hr while through movements have a saturation flow rate of 1900 veh/hr.

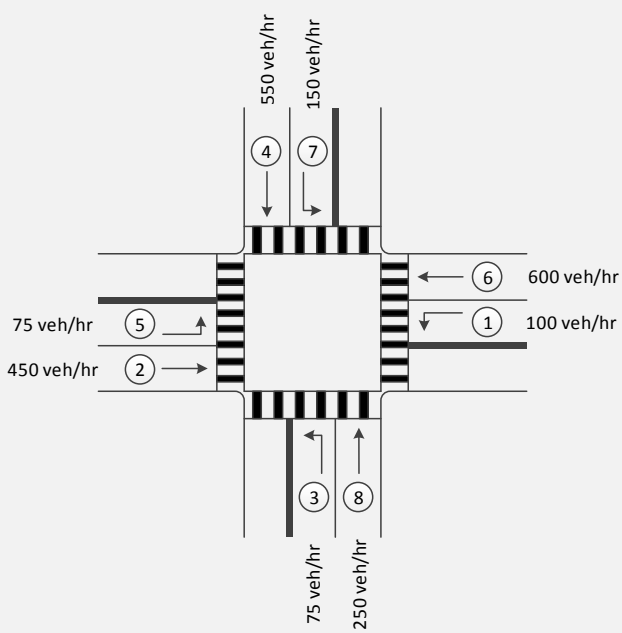


Figure 50. Intersection characteristics for Example Calculation 13

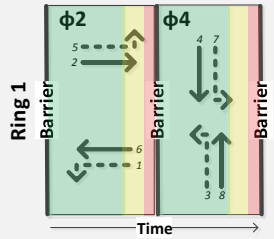


Figure 51. Ring barrier diagram for Example Calculation 13

Step 1: Compute the flow ratio Y_i for each movement i present at the intersection.
The flow ratios for each of the movements are calculated and shown in Figure 52.

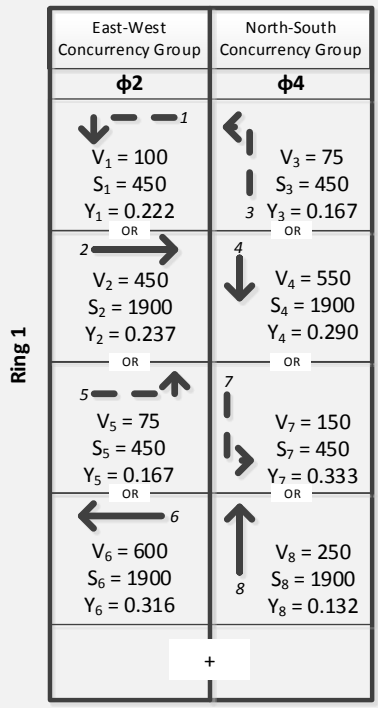


Figure 52. Flow ratios for each movement for, Example Calculation 13

Step 2: Determine the flow ratio sums for the phase sequences in each ring for each concurrency group (for the case of protected left turns only).
Since this example is for permitted left turns, we skip to step 3.

Step 3: Within each concurrency group, identify the movements with the maximum flow ratio sum (for protected left turns) or the movement with the maximum flow ratio (for permitted left turns). These movements are the critical movements for each concurrency group.
Since this example is for permitted left turns, we identify the movement with the maximum flow ratio in each concurrency group. As shown in Figure 53, movement 2 has the highest flow ratio (.316) for the east-west concurrency group. For the north-south concurrency group, movement 7 has highest flow ratio (.333).

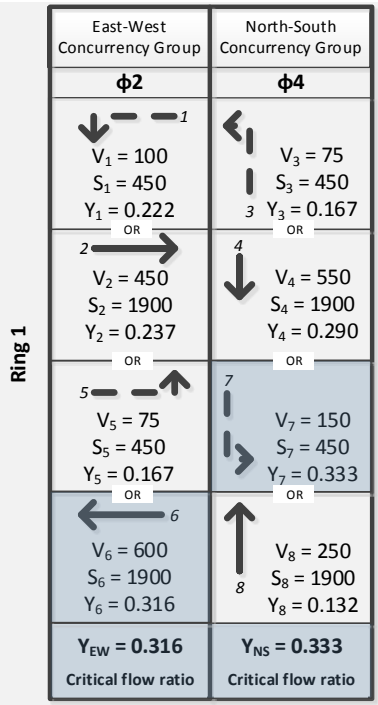


Figure 53. Critical flow ratios for Example Calculation 13

Step 4: Determine the critical volume-to-capacity ratio (X_c) for the intersection.
The critical flow ratios (Y_{EW}=.316, Y_{NS}=.333) were determined in step 3. The cycle length is given as 90 sec and the lost time per phase is 4 sec. There are two critical phases since all left turns are permitted, so the total lost time L is 8 sec. The critical volume-to-capacity ratio is computed using Equation 17.

$$X_c = \frac{(Y_{EW-critical} + Y_{NS-critical})(C)}{C - L} = \frac{(.316 + .333)(90\ sec)}{90\ sec - 8\ sec} = .71$$

Step 5: Based on the value of X_c calculated in step 4, determine the sufficiency of capacity.
The critical volume-to-capacity ratio, X_c = 0.71, indicates that the intersection is operating under capacity, using the ratings from Table 6.

8. Scenario 3. Calculating Uniform Delay of an Approach When Demand is less than Capacity

Just as we earlier asked the question "is there sufficient capacity at this intersection to accommodate demand", we can also ask another related question: "how is the intersection performing?" Or, stated another way: "what quality of service is provided to the users of the intersection?" Sometimes this question is asked for one intersection and its individual movements. Often it is asked when we are comparing alternative designs for an intersection or comparing the operation of several different intersections. Delay is performance measure commonly used to evaluate intersection performance. Delay is a measure that can be perceived by users, and its calculation is illustrated in this section. This calculation is illustrated for one intersection approach, using Scenario 3. This scenario, shown in Figure 54, is based on one through movement on one intersection approach, shown as movement 2. Pretimed signal control is assumed. Demand on the approach is assumed to be less than the capacity.

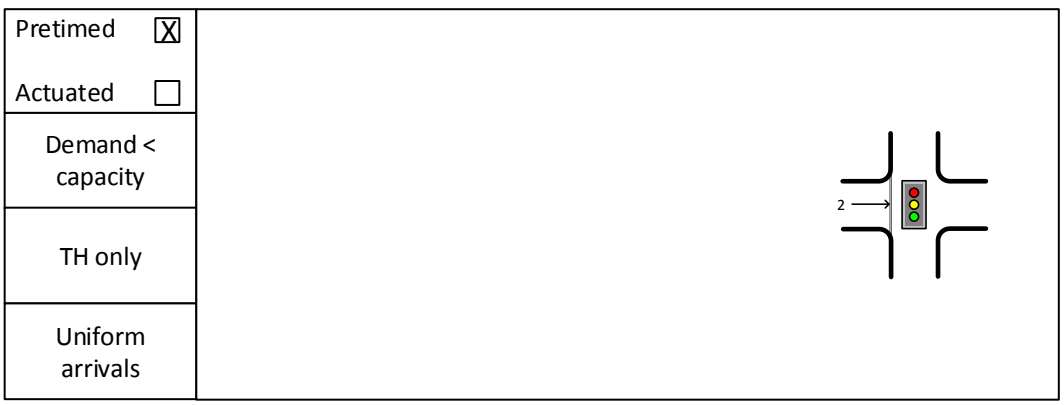


Figure 54. Scenario 3

Determining Uniform Delay Using the Cumulative Vehicle Diagram and the Queue Accumulation Polygon

In section 2, the cumulative vehicle diagram and the queue accumulation polygon were presented as two ways of representing traffic flow at a signalized intersection. Figure 55 shows a cumulative vehicle diagram highlighting the cumulative number of vehicles that have arrived at and departed from the signalized intersection over time. The slope of the cumulative arrival line (solid line) is equal to the arrival rate v. Three time periods are shown for the cumulative departure line:

- Effective red (period 1), during which the departure flow is zero,
- The queue service time g_s (period 2), in which the slope of the cumulative departure line is equal to the saturation flow rate s, and
- The final portion of effective green (period 3), in which the slopes of the cumulative arrival line and the cumulative departure line are equal to the arrival rate v.

The horizontal line connecting the arrival line and the departure line for each vehicle (shown in Figure 55 as d_i) is the delay experienced by that vehicle. The length of the queue, measured in vehicles, is the vertical distance at a given point in time between the arrival line and the departure line. The area of the triangle is equal to the total delay experienced by all vehicles that arrive during the cycle. The delay is called uniform delay since vehicles are assumed to arrive at the intersection at a uniform flow rate.

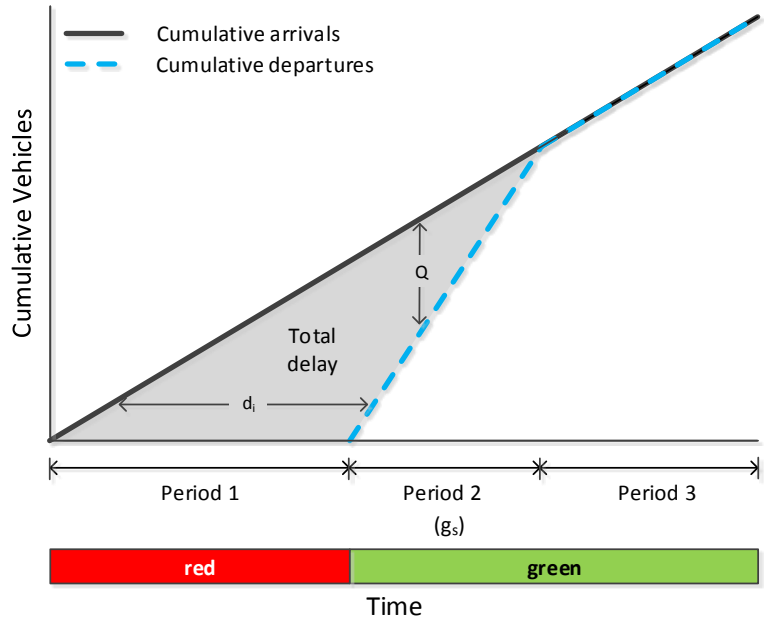


Figure 55. Cumulative vehicle diagram

Figure 56 shows a queue accumulation polygon for the same traffic flow conditions and time periods as shown in Figure 55. Here, the height of the polygon is the length of the queue at any point in time. The maximum queue length occurs at the end of the effective red interval. The area of the triangle is equal to the total delay experienced by all vehicles that arrive during the cycle.

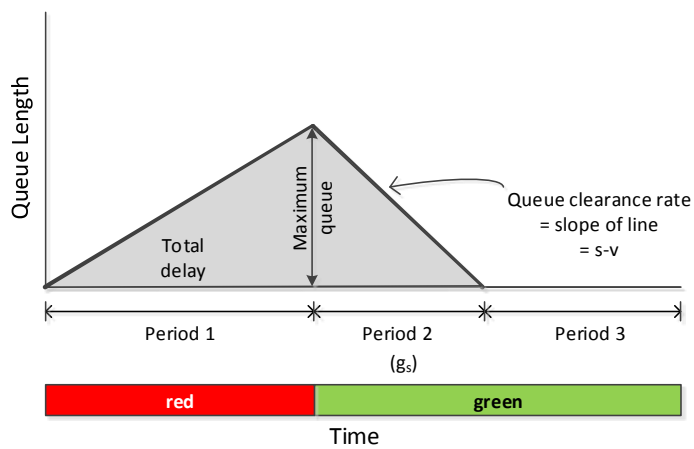


Figure 56. Queue accumulation polygon

Two assumptions made about the D/D/1 queuing model are represented in these diagrams:

- The queue clears before the end of the effective green, implying that the arrival volume is less than the capacity. This also implies that the queue at the beginning of red is zero.
- The arrival pattern is uniform.

The first step in determining the area of the triangle (total delay) is to compute the time that it takes for the queue to clear after the beginning of effective green. We call this time the queue

service time g_s. When the queue clears, the number of vehicles that have arrived at the intersection since the beginning of red must equal the number of vehicles that have departed since the beginning of the effective green. We can write this equality as

Equation 18

$$v(r + g_s) = sg_s$$

where

v = arrival rate, veh/sec,
r = effective red time, sec,
g_s = queue service time, sec, and
s = saturation flow rate, veh/sec.

Solving for g_s:

Equation 19

$$g_s = \frac{vr}{s - v}$$

Stated in words, the queue service time is equal to the length of the queue at the end of effective red (*vr)* divided by the rate of queue clearance after the start of effective green (*s-v*).

The area of the triangle in Figure 55 is equal to one-half the product of the effective red time (the base of the triangle) and the number of vehicles that have arrived at the intersection at the point that the queue has cleared, $v(r + g_s)$. This latter number is the height of the triangle. The total uniform delay D_t is given in Equation 20.

Equation 20

$$D_t = (0.5)(r)[(v)(r + g_s)]$$

where the variables are defined as above. Substituting g_s from Equation 19, we get another expression for the total delay.

Equation 21

$$D_t = (0.5)(r)\left[(v)\left(r + \frac{vr}{s - v}\right)\right]$$

Equation 22

$$D_t = \frac{(0.5vr)rs}{s - v}$$

Equation 23

$$D_t = \frac{0.5vr^2}{1 - v/s}$$

The number of vehicles that arrive during the cycle is the product of the arrival rate v and the cycle length C. The *average uniform delay per vehicle* d_{avg} is the total delay from Equation 21 divided by the number of vehicles that arrive during the cycle (*vC)*.

Equation 24

$$d_{avg} = \frac{(0.5)(r)}{vC}\left[(v)\left(r + \frac{vr}{s - v}\right)\right]$$

When terms are rearranged and simplified:

Equation 25

$$d_{avg} = (0.5r)\left[\frac{(1 - g/C)}{(1 - v/s)}\right]$$

The same equation will result if we compute the area from the queue accumulation polygon in Figure 56. These equations yield the average delay experienced by vehicles if the arrival pattern is uniform and if the demand is less than the capacity of the approach. When we examine the equation for average delay, we can see that delay increases when:

- The effective red increases, resulting in a longer queue that forms during red.
- The effective green ratio decreases, providing less green time during the cycle to serve the queue.
- The flow ratio increases, as the volume approaches the capacity.

The average uniform delay for the entire intersection can also be computed. If the delay for each approach is computed to be d_i and the volume on each approach is v_i, the average delay for the intersection d_{int} is the weighted average of the delays for each of the intersection approaches.

Equation 26

$$d_{int} = \frac{\sum d_i v_i}{\sum v_i}$$

where

d_i = delay for each of the i approaches at the intersection, sec, and

v_i = the volume for each of these approaches, veh/sec.

Example Calculation 14. Calculation of Average Delay When Volume Is Less than Capacity

An intersection approach has an arrival rate of 630 veh/hr and a saturation flow rate of 1900 veh/hr. The cycle length is 100 sec, the effective red time is 60 sec, and the effective green time is 40 sec. Determine the queue service time and the average delay for this approach.

Step 1. Convert the flow rates from veh/hr to veh/sec.

$$v = \frac{630\ veh/hr}{3600\ sec/hr} = 0.175\ veh/sec$$

$$s = \frac{1900\ veh/hr}{3600\ sec/hr} = 0.528\ veh/sec$$

Step 2. Using Equation 3, calculate the approach capacity and compare it to the arrival flow rate.

$$c = s \times \left(\frac{g}{C}\right) = 1900\ veh/hr\ \times \left(\frac{40\ sec}{100\ sec}\right) = 760\ veh/hr$$

The arrival volume (630 veh/hr) is thus less than the capacity (760 veh/hr), so the analytical method (Equation 25) can be used to calculate the average uniform delay.

Step 3. Calculate the queue service time using Equation 19.

$$g_s = \frac{vr}{s - v} = \frac{(.175\ veh/sec)(60\ \text{sec})}{.528\ veh/sec - .175\ veh/sec} = 29.7\ sec$$

Step 4. Construct the cumulative vehicle diagram and the queue accumulation polygon. (Though this step is not necessary to solve this problem, the preparation of the two diagrams is recommended for better understanding these concepts).

Cumulative vehicle diagram: The arrival line (solid line, Figure 57) shows the cumulative number of vehicles that arrive from the beginning of the cycle (t = 0) to the end of the cycle (t = 100). The departure line (dashed line) shows the cumulative number of departures over time. During the effective red (from t = 0 to t = 60), the number of departures is zero. At the end of the effective red, the queue is 10.5 vehicles (line 1-2). From the beginning of effective green to the time that the queue clears (point 3), the departure line has a slope equal to the saturation flow rate. After the queue has cleared (t = 89.7, 29.7 sec after the beginning of effective green), the arrival line and the departure line are coincident.

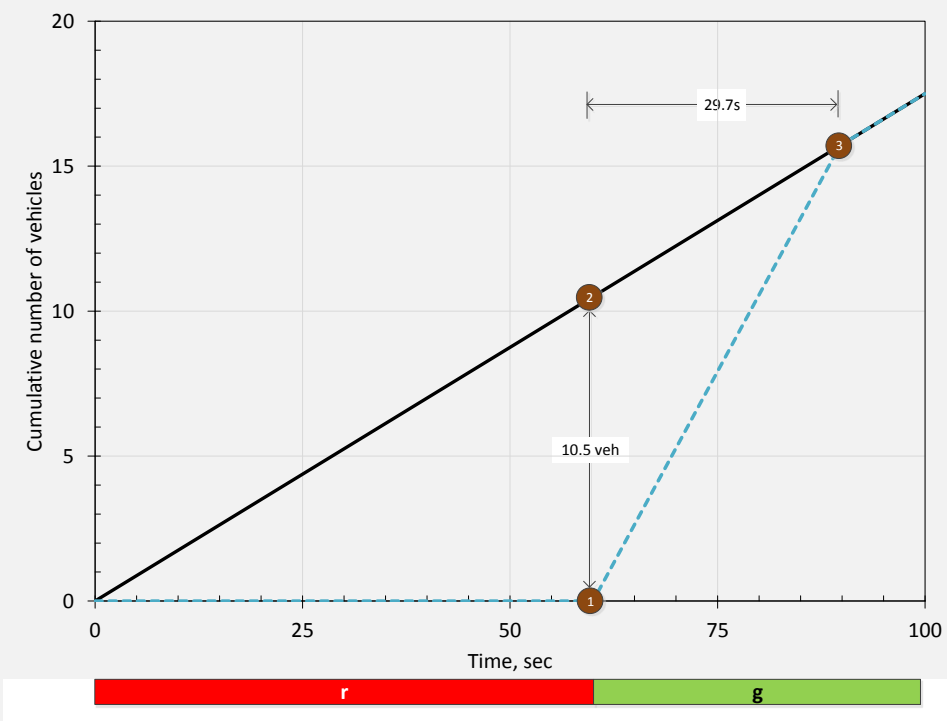


Figure 57. Cumulative vehicle diagram for Example Calculation 14

Queue accumulation polygon: The height of the polygon in Figure 58 shows the number of vehicles in the queue at any point during the cycle.

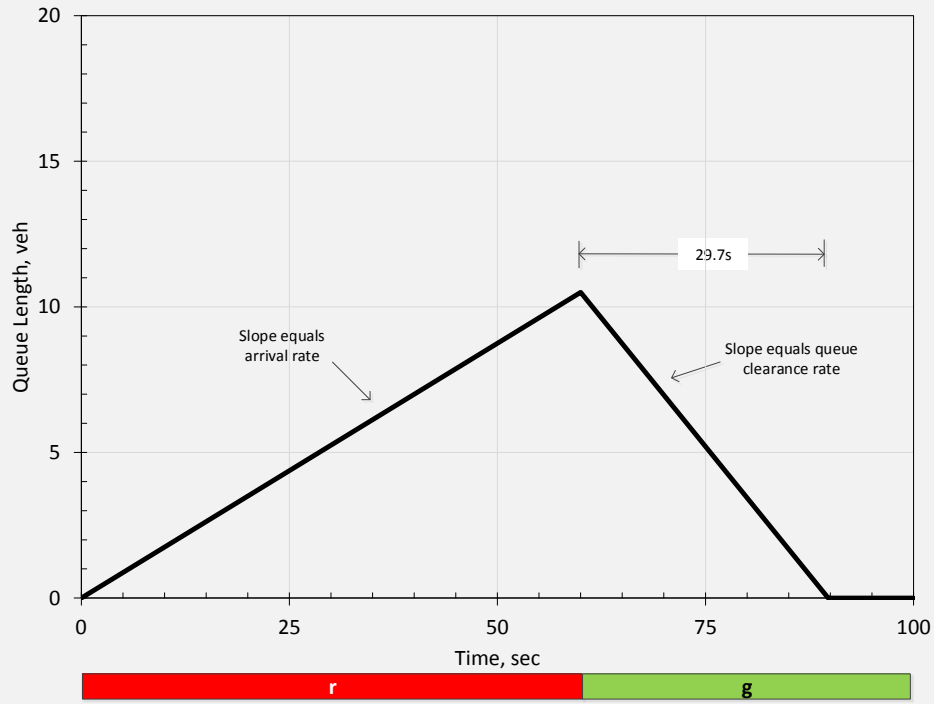


Figure 58. Queue accumulation polygon for Example Calculation 14

Step 5. The average delay is calculated using Equation 25:

$$d_{avg} = (0.5r)\left[\frac{\left(1-\frac{g}{C}\right)}{\left(1-\frac{v}{s}\right)}\right]$$

$$d_{avg} = (0.5 \times 60\ sec)\left[\frac{\left(1-\frac{40\ sec}{100\ sec}\right)}{\left(1-\frac{630\ veh/hr}{1900\ veh/hr}\right)}\right] = 26.9\ sec$$

9. Scenario 4. Calculating Delay When Demand Exceeds Capacity

When the demand on an intersection approach exceeds its capacity, we can't use the uniform delay equation developed in section 8 as part of Scenario 3. When the demand exceeds capacity, a basic assumed of the D/D/1 queuing model is violated and we must use a graphical method to compute delay. In Scenario 4, as illustrated in Figure 59, we calculate the delay of an intersection approach using the cumulative vehicle diagram and the queue accumulation polygon. We calculate the delay based on the areas of the polygons in the cumulative vehicle diagram and the queue accumulation polygon. The graphical method is developed in Example Calculation 15.

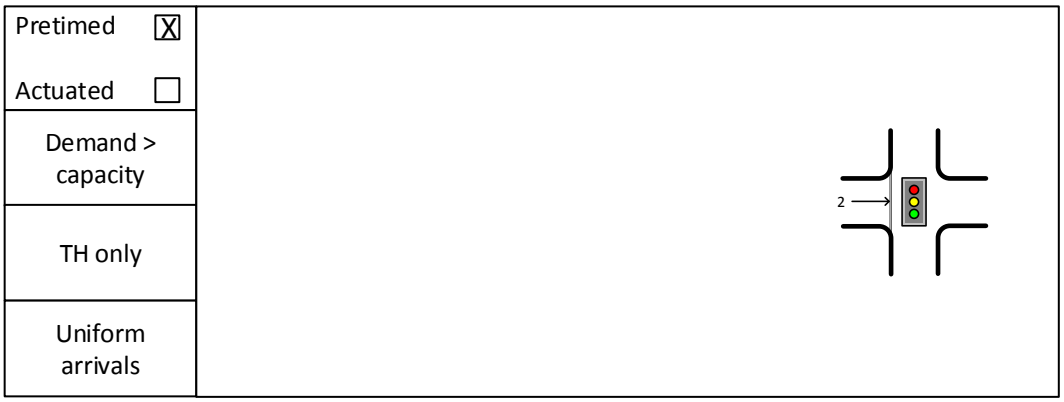


Figure 59. Scenario 4

Example Calculation 15. Calculation of Average Delay When Volume Exceeds Capacity
An intersection approach has a cycle length of 100 sec and an effective green time of 40 sec. The arrival rate varies over three cycles. The arrival rate is 900 veh/hr during the first cycle, 720 veh/hr during the second cycle, and 540 veh/hr during the third cycle. Calculate the average delay for the approach over the three cycles. The saturation flow rate is 1900 veh/hr.

Step 1. Convert the flow rates from veh/hr to veh/sec. The resulting rates for each cycle are shown in Table 7.

Table 7. Vehicle arrival rates during each cycle

Cycle	Flow rate (veh/hr)	Flow rate (veh/sec)
1	900	.25
2	720	.20
3	540	.15

The flow profile diagram is shown in Figure 60.

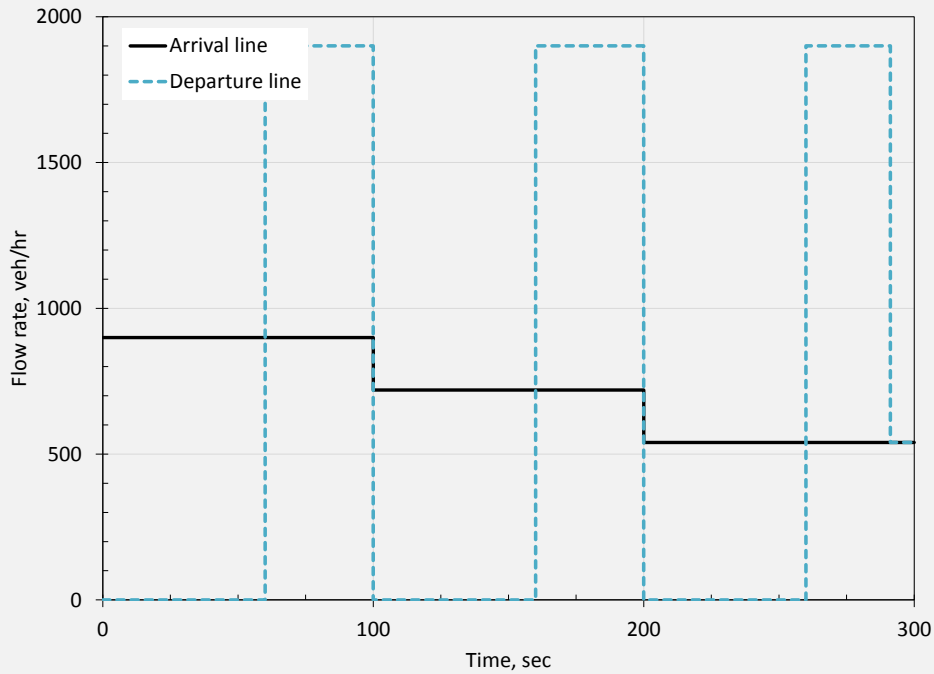


Figure 60. Flow profile diagram for Example Calculation 15

Step 2. Using Equation 5, calculate the approach capacity and compare it to the arrival flow rate for each cycle.

$$c = s \times \left(\frac{g}{C}\right) = 1900\ veh/hr\ \times \left(\frac{40\ sec}{100\ sec}\right) = 760\ veh/hr$$

The arrival flow rate is less than the capacity for the second and third cycle. But in the first cycle, the volume of 900 veh/hr exceeds the capacity of 760 veh/hr. Because volume exceeds capacity, Equation 25 cannot be applied to determine the delay. We must instead use the graphical approach of the cumulative vehicle diagram or queue accumulation diagram.

Step 3. Calculate the queue service time g_s for the first cycle using Equation 19.

$$g_s = \frac{vr}{s - v} = \frac{(.250\ veh/sec)(60\ \text{sec})}{.528\ veh/sec - .250\ veh/sec} = 54.0\ sec$$

This result confirms the finding in step 2 that the volume exceeds capacity. The queue would take 54.0 sec to clear, longer than the effective green time of 40 sec.

Step 4. Construct the cumulative vehicle diagram and queue accumulation polygon for the three cycles using the flow rate data given above.

Cumulative vehicle diagram: The arrival line (solid line, Figure 61) shows the cumulative number of vehicles that arrive from the beginning of the first cycle (t=0) to the end of the third cycle (t=300). The departure line (dashed line) shows the cumulative number of departures over time. The number of vehicles in queue at the end of each of the effective red and effective green periods are shown by the vertical lines.

Queue accumulation polygon. Figure 62 shows the variation in the queue length during the three cycles. The queue does not clear by the end of the first cycle, consistent with the calculations shown in steps 2 and 3. The residual queue at this point is 3.9 vehicles. Although the volume is less than the capacity in the second cycle, it is not sufficiently less to allow the queue to clear. There is still a residual queue of 2.8 vehicles at the end of cycle 2.

During the third cycle, the queue finally clears. But since there is a residual queue at the end of the second cycle, the numerator of Equation 19 (used to calculate the queue service time) must include both this residual queue and the queue that forms during effective red in the third cycle. The queue clears 31.2 seconds after the beginning of the effective green.

$$g_s = \frac{(residual\ queue) + vr}{s - v} = \frac{2.8\ veh + (.15\ veh/sec)(60\ sec)}{(.528\ veh/sec - .150\ veh/\text{sec})}$$

$$g_s = 31.2\ sec$$

Step 5. Determine the total delay. The total delay can be calculated as the area under the curve in the cumulative vehicle diagram or queue accumulation polygon. Here we will use the queue accumulation polygon, divided into six separate polygons to facilitate the calculation of the area as shown in Figure 63.

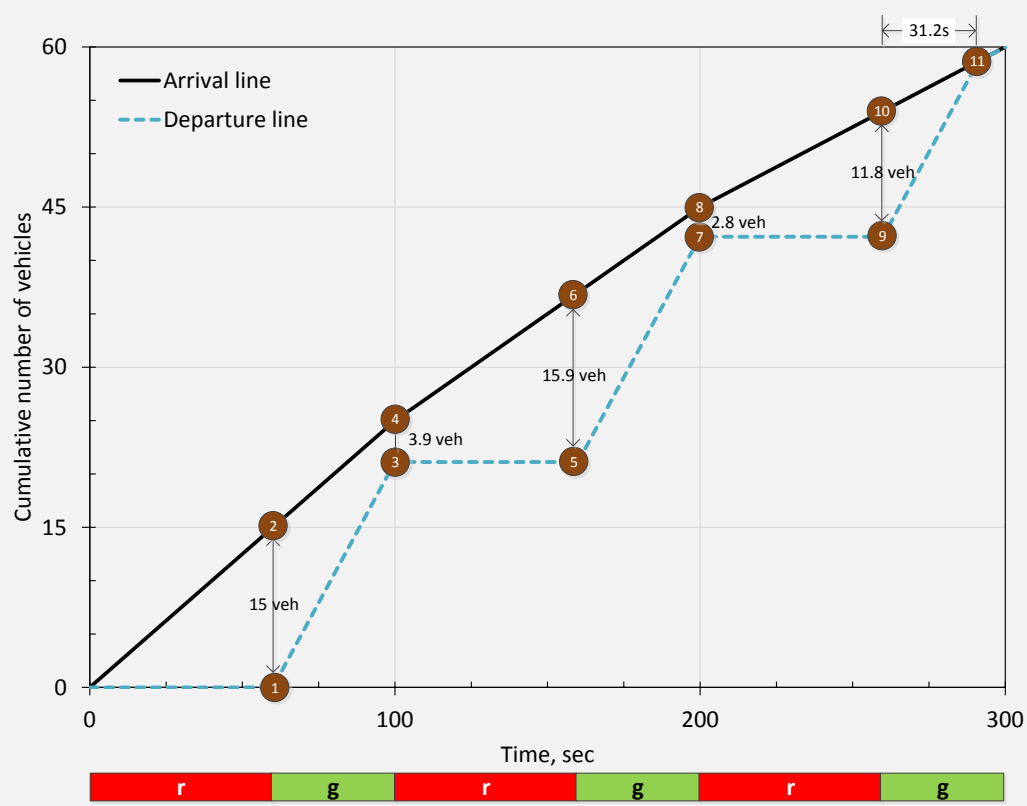


Figure 61. Cumulative vehicle diagram for Example Calculation 15

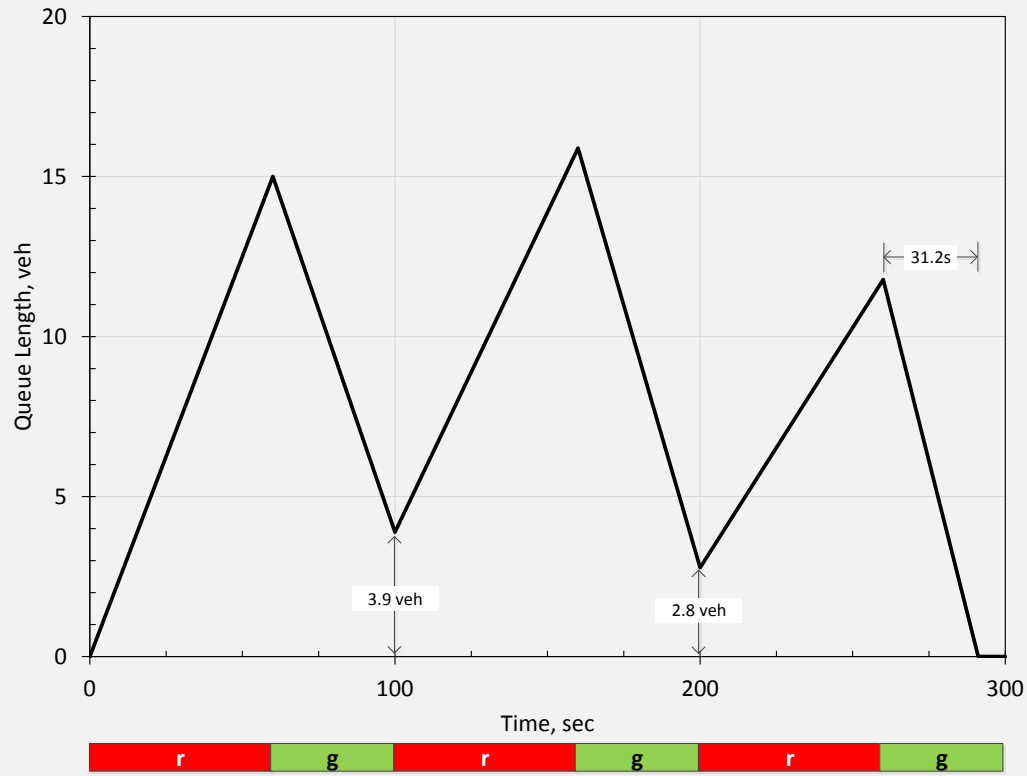


Figure 62. Queue accumulation polygon for Example Calculation 15

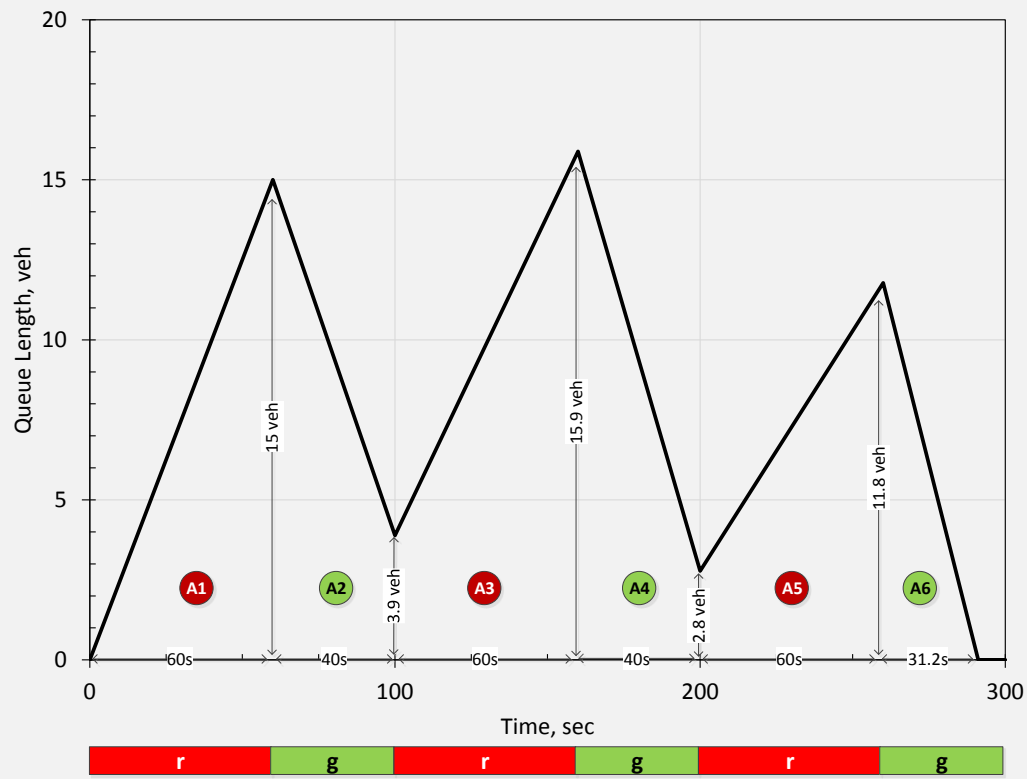


Figure 63. Constituent polygons from queue accumulation polygon, Example Calculation 15

The calculations to determine the areas of each of the six constituent polygons are shown below.

Cycle 1:

$$A1 = \frac{15\ veh}{2} \times (60\ sec) = 450\ veh - sec$$

$$A2 = \frac{3.9\ veh + 15\ veh}{2} \times (100\ sec - 60\ sec) = 378\ veh - sec$$

Cycle 2:

$$A3 = \frac{15.9\ veh + 3.9\ veh}{2} \times (160\ sec - 100\ sec) = 594\ veh - sec$$

$$A4 = \frac{2.8\ veh + 15.9\ veh}{2} \times (200\ sec - 160\ sec) = 374\ veh - sec$$

Cycle 3:

$$A5 = \frac{11.8\ veh + 2.8\ veh}{2} \times (260\ sec - 200\ sec) = 438\ veh - sec$$

$$A6 = \frac{11.8\ veh}{2} \times (291.2\ sec - 260\ sec) = 184\ veh - sec$$

The sum of these areas is the total delay for three cycles.

$$D_t = A_1 + A_2 + A_3 + A_4 + A_5 + A_6$$

$$D_t = 2418\ veh - sec$$

Step 6. Determine the number of vehicles that arrive during the three cycles. Based on the arrival rates given in Table 7 and the length of the cycle (*C* = 100 sec), the total number of vehicles arriving during the three cycles is calculated below. Note that the number of vehicle arrivals can also be determined from the cumulative vehicle diagram in Figure 61.

Cycle 1:

$$Vehicle\ arrivals = (.25\ veh/sec)(100\ sec) = 25\ \ veh$$

Cycle 2:

$$Vehicle\ arrivals = (.20\ veh/sec)(100\ sec) = 20\ \ veh$$

Cycle 3:

$$Vehicle\ arrivals = (.15\ veh/sec)(100\ sec) = 15\ \ veh$$

The total arrivals during three cycles:

$$Vehicle\ arrivals = 25\ veh + 20\ veh + 15\ veh = 60\ \ veh$$

Step 7. Calculate average delay.

$$d_{avg} = \frac{D_t}{number\ vehicles\ arrived} = \frac{2418\ veh - sec}{60\ veh} = 40.3\ sec$$

10. Scenario 5. Calculating the Capacity of an Exclusive LT Lane with Permitted LT Phasing

When a permitted left turn movement is opposed by a TH movement, we know that its capacity is less than if the movement was protected. To illustrate the calculation of a permitted LT movement we use Scenario 5. In this scenario, as illustrated in Figure 64, we consider a permitted LT movement from an exclusive LT lane (movement 5 in the figure) opposed by a TH movement from a single lane (movement 6 in the figure).

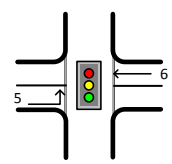
Pretimed ☒ Actuated ☐	
Demand < capacity	
Permitted LTs	
Uniform arrivals	

Figure 64. Scenario 5

Let's look first when we have protected LT phasing and an exclusive LT lane. What is the capacity of the LT movement? We know from Equation 3 that the capacity of a lane or approach is:

$$c = s\left(\frac{g}{C}\right)$$

However, the saturation flow rate for a LT movement is lower than for a through movement. LT vehicles, even with protected LT phasing, depart from the intersection more slowly than through vehicles. The intersection geometry often restricts the speed of a LT vehicle as well. Research has shown [xx] that this lower speed results in a saturation flow rate that is five percent lower than the idea rate of 1900 veh/hr.

Example Calculation 16. Calculating the Capacity of a Permitted LT Lane

Suppose that the green ratio for a protected LT movement operating from an exclusive LT lane is 0.5. What is the capacity of this lane?

We know from the discussion above that the saturation flow rate is 5 percent lower than the ideal rate of 1900 veh/hr. We can calculate the capacity of the movement using Equation 3, modified with this reduction of 0.95.

$$c = s\left(\frac{g}{C}\right) = (0.95)\left(1900\,\frac{veh}{hr}\right)(0.95) = 902\ veh/hr$$

Now consider another case, this time an exclusive LT lane controlled by permitted phasing. We can represent the flow patterns for one cycle using the three queuing diagrams, the flow profile diagram, the cumulative vehicle diagram, and the queue accumulation polygon.

Figure 65 shows the flow profile diagram for the arrival flow and the departure flow. The arrival flow rate is uniform. The departure flow rate is zero for the red interval. But it is also zero for the first part of the green interval as the queue for the opposing through movement clears. Once the opposing queue clears, during the remainder of the green interval, the LT vehicles enter the intersection as they find usable gaps in the opposing traffic stream. Figure 66 shows the cumulative vehicles that arrive and depart during one cycle, based on the flow profile diagram from Figure 65. Figure 67 shows the resulting queue accumulation polygon. Both the cumulative vehicle diagram and the queue accumulation polygon as shown here assume that the queue for the permitted LT movement clears at the end of green.

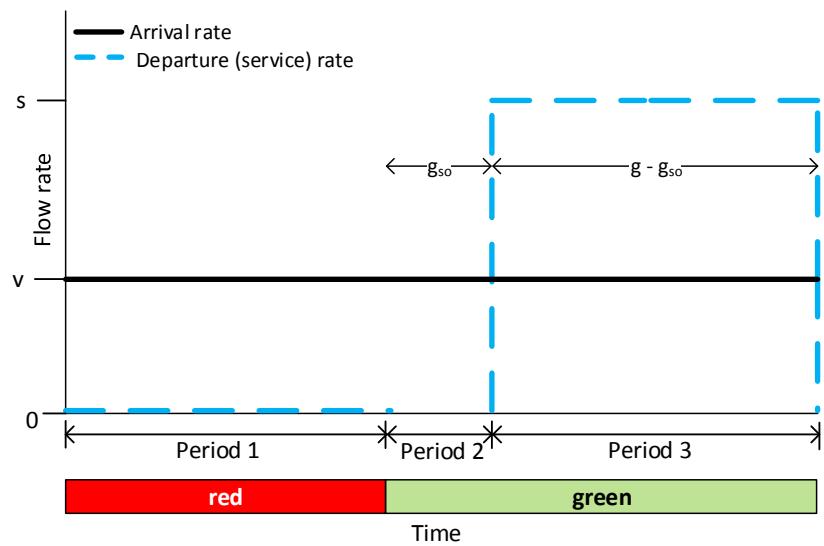


Figure 65. Flow profile diagram for Example Calculation 16

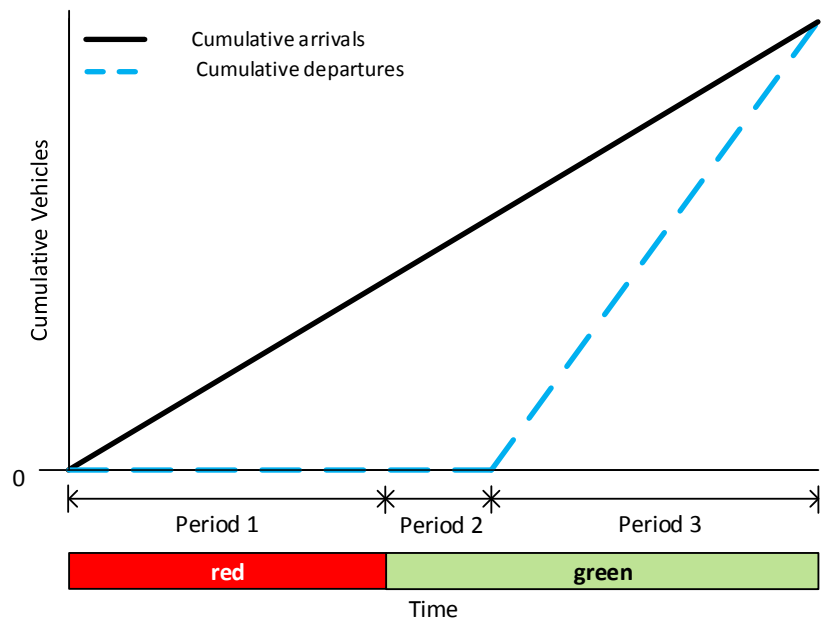


Figure 66. Cumulative vehicle diagram for Example Calculation 16

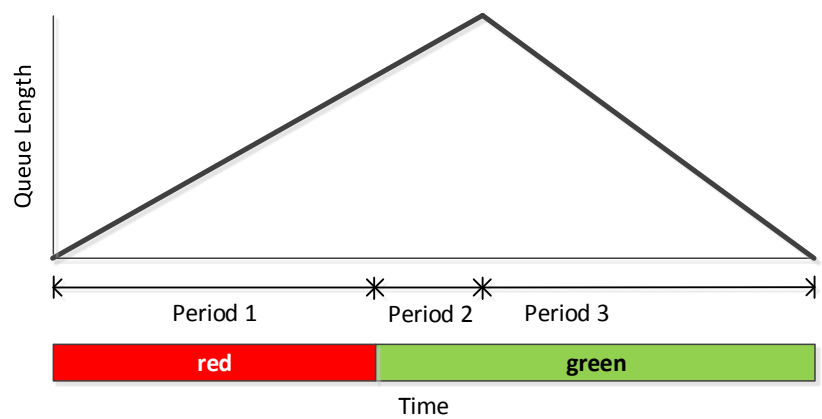


Figure 67. Queue accumulation polygon for Example Calculation 16

The saturation flow rate for LT vehicles during the period after the opposing queue has cleared is determined using the same model used for a non-priority movement at a TWSC intersection as given in Chapter 5. This flow rate, given by Equation 27, is:

Equation 27

$$s_p = \frac{v_o e^{-v_o t_c/3600}}{1 - e^{-v_o t_f/3600}}$$

where

s_p = saturation flow rate of a permitted left-turn movement (veh/h/ln);
v_o = opposing demand flow rate (veh/h);
t_c = critical headway = 4.5 (s); and
t_f = follow-up headway = 2.5 (s).

Equation 28 gives the duration of the green interval that is blocked by the clearing of the opposing queue, as per the discussion in section 8.

Equation 28

$$g_{so} = \frac{v_o r}{s - v_o}$$

where

g_{so} = the time for the opposing queue to clear (sec)
v_0 = opposing demand flow rate (veh/hr)
r = the duration of the effective red interval (sec)
s = saturation flow rate of the opposing TH movement (veh/hr)

The capacity of the LT movement is the calculated as the product of the saturation flow rate s_p and the green ratio for the green interval after the opposing queue has cleared, g_{so}:

Equation 29

$$c = s_p \left(\frac{g_{so}}{C}\right)$$

Example Calculation 17. Calculating the Capacity of a Permitted LT movement from an Exclusive LT lane

Figure 68 shows two movements, the NBLT movement (controlled by permitted LT phasing) and the opposing TH (SB) movement. The cycle length is given as 60 sec, the green ratio for the NB/SB movements is 0.5, and the ideal saturation flow rate for the SBTH movement is 1900 veh/hr. What is the capacity of the LT lane?

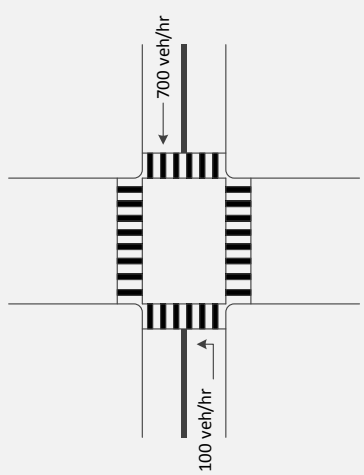


Figure 68. Volume data, Example Calculation 17

Step 1. Prepare a sketch of the departure flow profile and the queue accumulation diagram for both the NB and SB approaches. Identify the key time intervals for one cycle for both approaches. Figure 69 shows the flow profile diagram and the queue accumulation polygon for the SB TH movement.

The queue service time for the SB movement is calculated using Equation 28:

$$g_{so} = \frac{v_o r}{s - v_o} = \frac{(700\ veh/hr)(30\ \text{sec})}{(1900\ veh/hr\ \ - 700\ veh/hr)} = 17.5\ sec$$

From t = 30 sec to t – 47.5 sec, the clearing SB queue prevents any NBLT vehicles from entering the intersection.

Step 2. Calculate the departure flow rates for the NB movement.

- During red, the NB flow is zero.
- During the time that the opposing queue is clearing, the NB flow is zero (g_{so}).
- After the opposing queue has cleared, the NBLT vehicles can filter through the opposing SB flow, as acceptable gaps appear.

The saturation flow rate for this third time interval is calculated using Equation 27:

$$s_p = \frac{v_o e^{-v_o t_c/3600}}{1 - e^{-v_o t_f/3600}} = \frac{700 e^{-(700)(4.5)/3600}}{1 - e^{-(700)(25)/3600}} = \frac{291.8}{.385} = 758\ veh/hr$$

This is the saturation flow rate for this approach. The capacity is the product of this saturation flow rate and the green ratio for the green interval after the opposing queue has cleared:

$$c = s_p\left(\frac{g_{so}}{C}\right) = \frac{(758\ veh/hr)(12.5\ \text{sec})}{60\ sec} = 158\ veh/hr$$

The only time interval during which the NBLT movement can move is during g_{so}, and only in permitted mode.

Figure 70 shows the flow profile diagram and the queue accumulation polygon for the NB LT movement. The NBLT movement only begins to move after the opposing SB queue has cleared, at t = 47.5 sec.

As a comparison, suppose that the green ratio for a protected LT movement is the same as calculated above for the permitted LT movement. What is the resulting capacity? Assume that the green ratio for the NB/SB approaches is 0.5 and the resulting green time is 30 sec. The available green time during the permitted period is thus 30 sec – 12.5 sec = 17.5 sec.

$$c = s\left(\frac{g}{C}\right) = (0.95)(1900\ veh/hr)\left(\frac{17.5\ sec}{60\ sec}\right) = 526\ veh/hr$$

Thus the capacity for a protected LT movement is more than three times that of a permitted LT movement for the conditions given in this example.

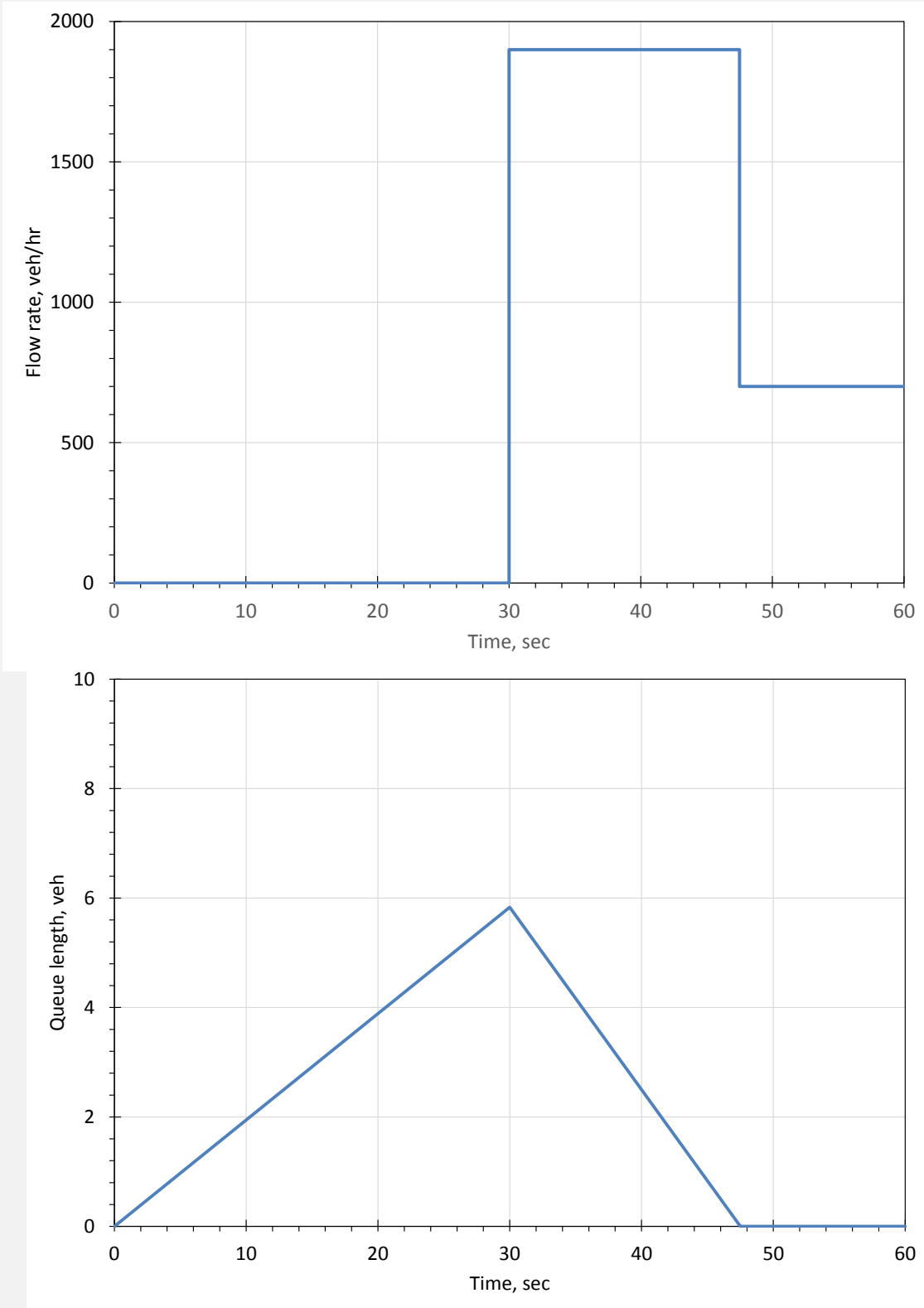
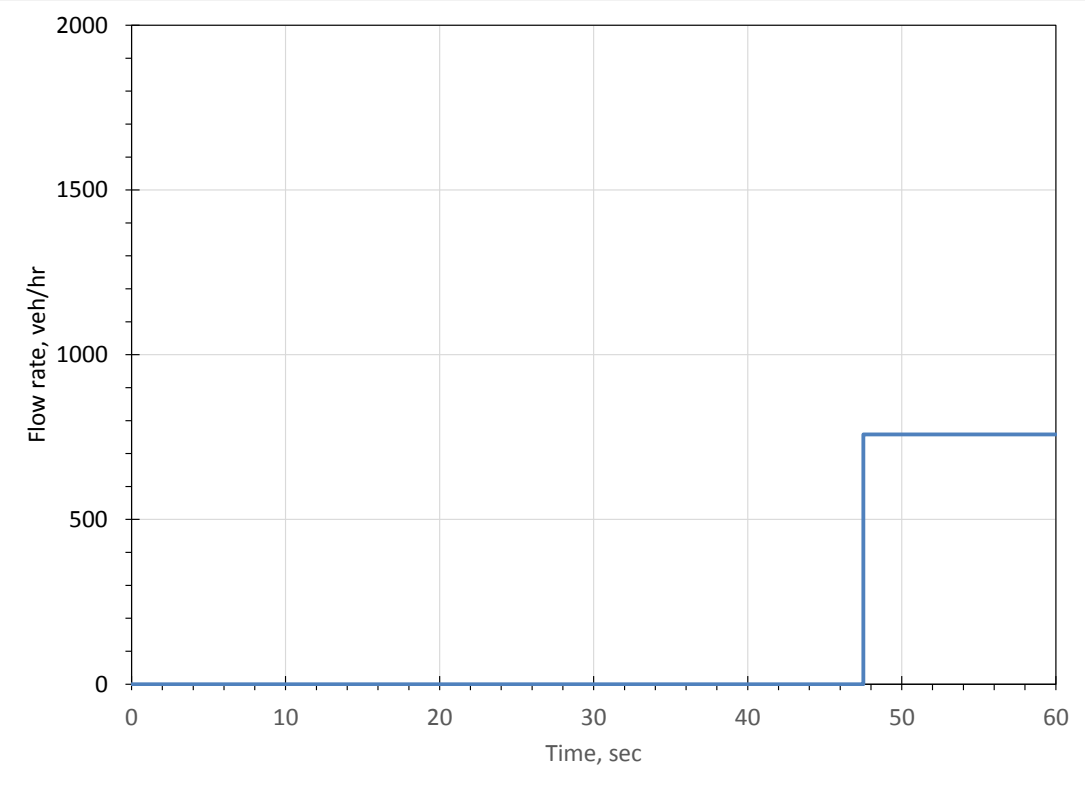


Figure 69. Flow profile diagram and queue accumulation polygon for SB movement, Example Calculation 17



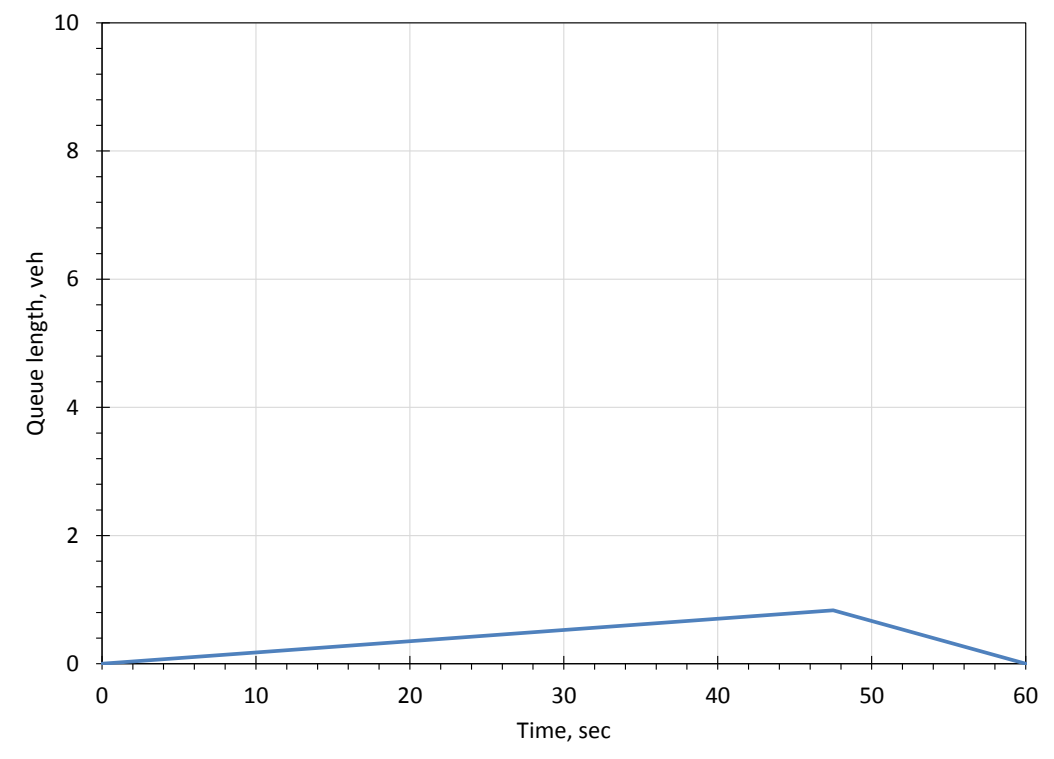


Figure 70. Flow profile diagram and queue accumulation polygon for NB movement, Example Calculation 17

11. Scenario 6. Calculating Delay When the Arrival Pattern is Non-Uniform

Often the arrival pattern of vehicles at a signalized intersection is not uniform. This can occur when a signalized intersection is located upstream and produces a series of platoons arriving at the downstream intersection rather than the uniform arrival pattern that we've previously assumed. This arrival pattern is a function of the distance between the intersections, the travel speed of vehicles, and the relative offset between the intersections. The offset is the time that the green interval begins at the downstream intersection with respect to the start of green at the upstream intersection.

We illustrate the calculation of the delay on an approach to a signalized intersection when the arrival pattern is not uniform using Scenario 6. We consider movement 2, a TH movement on a one-lane approach to the downstream intersection, shown on the right in Figure 71. Movement 2 at the upstream intersection feeds movement 2 arriving at the downstream intersection. The signal control is pretimed. We consider how the predicted arrival flow profile overlays onto the time sequence of the red and green intervals serving movement 2.

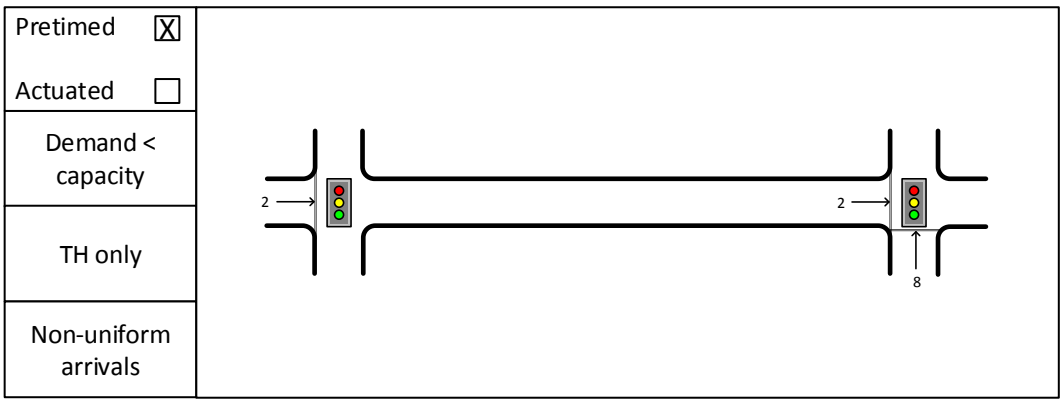


Figure 71. Scenario 6

Platoon Dispersion Model

We represent this arrival flow pattern with a platoon dispersion model, developed by Robertson [xx] and modified by Bonneson [xx] and others [xx]. Given a platoon departing from an upstream signalized intersection, the model predicts the flow arriving some distance away at a downstream signalized intersection. This model considers the dispersion of the tightly packed platoon departing from the upstream intersection, reflecting the different speeds the various drivers choose as they travel along the arterial toward the downstream intersection. The headways between vehicles in a queue clearing at an upstream intersection vary around the saturation headway, but this variance is small. However, the variance of the headways in a dispersing platoon is much larger as drivers choose their own following distances and travel speeds.

The platoon dispersion model (represented in Figure 72 using a flow profile diagram) shows that the predicted flow for a one-second time step (shown in area 1) as a function of the flows in two earlier time steps:

- the departure flow from the upstream intersection t time steps earlier (represented by area 3) and,
- the arrival flow at the downstream intersection one time step earlier (represented by area 2).

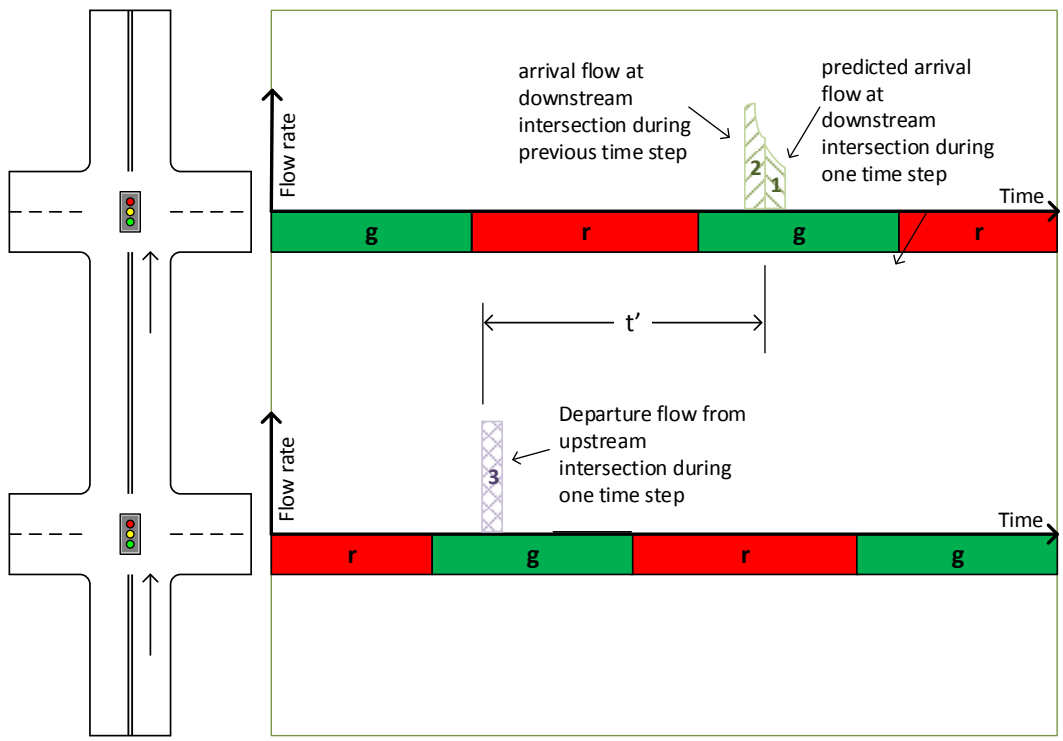


Figure 72. Platoon dispersion model, Scenario 6

Mathematically, we represent the model as shown in Equation 30. The predicted flow at the downstream intersection during time step j is a function of the flow departing from the upstream intersection t time steps earlier and the flow at the downstream intersection one time step earlier.

Equation 30

$$q_{d,j} = Fq_{u,j-t} + (1 - F)q_{d,j-1}$$

where

$q_{d,j}$ = the arrival flow rate in time step j at the downstream intersection,
$q_{u,j-t}$ = the departure flow rate in time step j-t from the upstream intersection, and
$q_{d,j-1}$ = the arrival flow in time step j-1 at the downstream intersection.

F is a smoothing factor as given by Equation 31.

Equation 31

$$F = \frac{1}{1.315 + 0.138t_R}$$

The front of the platoon arrives at the downstream intersection t' seconds after it leaves the upstream interval as given by Equation 32.

Equation 32

$$t' = t_R - \frac{1}{F} + 1.25$$

Figure 73 shows a platoon leaving the upstream intersection and arriving at the downstream intersection. The flow rate at the downstream intersection is lower and the duration of the platoon longer than the platoon departing from the upstream intersection. The travel time that it takes for the front of the upstream platoon to reach the downstream intersection is also shown in the figure. Note also that the areas under both flow profile curves are equal, as the product of the flow rate and the duration of the platoon is equal to the number of vehicles in the platoon.

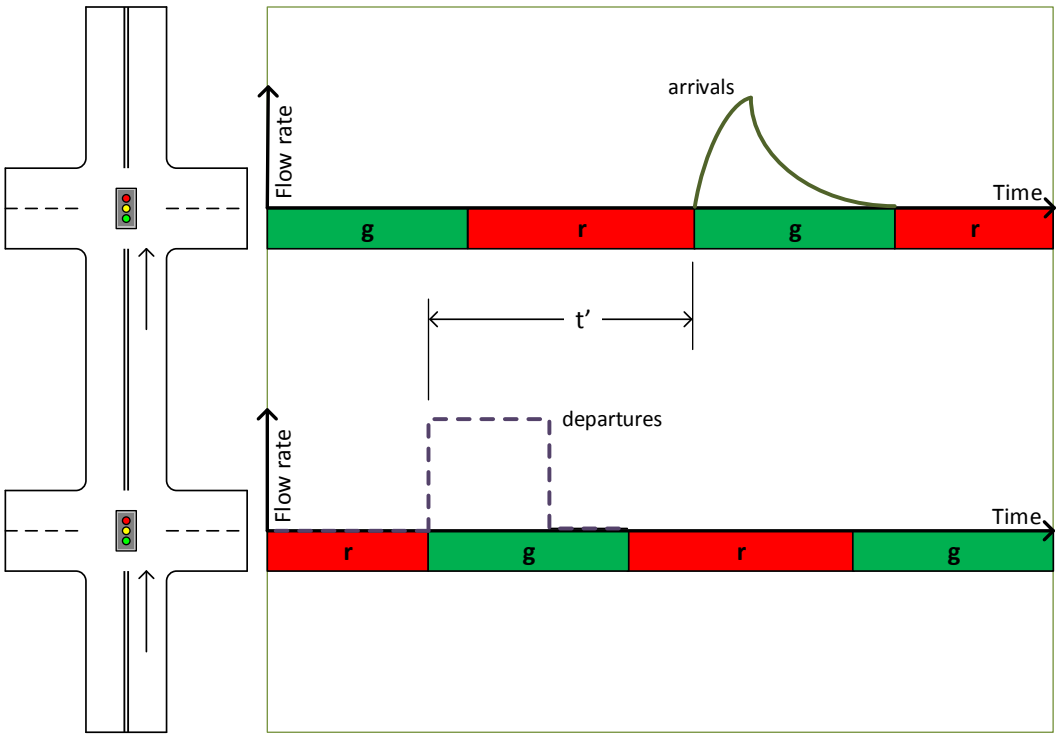


Figure 73. Platoon dispersion model, Scenario 6

Example Calculation 18. Calculating the Arrival Flow at the Downstream Intersection

Let's illustrate how the model works with an example calculation. Suppose two signalized intersections are located 1000 ft apart. Data for both signalized intersections include:

- C = 60 sec
- g/C = 0.5
- s = 1900 veh/hr

The average travel speed on the arterial is 25 mi/hr, while the arrival flow rate at the signalized intersection is 600 veh/hr.

The time that it takes for the queue to clear at the upstream signalized intersection (or the queue service time, g_s) is given by:

$$g_s = \frac{vr}{s - v} = \frac{(600\ veh/hr)(30\ sec)}{1900\ veh/hr - 600\ veh/hr} = 13.8\ sec$$

The running time from the upstream intersection to the downstream intersection is the segment length divided by the average travel speed. For this example:

$$t_R = \frac{d}{S} = \frac{1000\ ft}{(25\ mi/hr)(1.47)} = 27.2\ sec$$

The smoothing factor F (from Equation 31) is:

$$F = \frac{1}{1.315 + 0.138t_R} = \frac{1}{1.315 + (.138)(23.4)} = .197$$

The time for the platoon to reach the downstream intersection (from Equation 32) is given by:

$$t' = t_R - \frac{1}{F} + 1.25 = 27.2\,\text{sec} - \frac{1}{.197} + 1.25 = 23.4\ sec$$

Table 8 shows the departure flow rates from the upstream intersection and the arrival flow rates for the downstream intersection for each one second time step over a 30 second period, from t = 30 to t = 59. For the upstream intersection, the saturation flow rate is 1900 veh/hr for the 13.8 seconds that it takes for the queue to clear after the beginning of green. For the downstream intersection, we use Equation 30 to calculate the flow rates for each time step.

Table 8. Given upstream flow rates and predicted downstream flow rates, Example Calculation 18

Time (sec)	Upstream flow rate (veh/hr)	Downstream flow rate (veh/hr)
30	1900	0
31	1900	0
32	1900	0
33	1900	0
34	1900	0
35	1900	0
36	1900	0
37	1900	0
38	1900	0
39	1900	0
40	1900	0
41	1900	0
42	1900	0
43	1900	0
44	600	0

Time (sec)	Upstream flow rate (veh/hr)	Downstream flow rate (veh/hr)
45	600	0
46	600	0
47	600	0
48	600	0
49	600	0
50	600	0
51	600	0
52	600	0
53	600	0
54	600	375
55	600	676
56	600	917
57	600	1111
58	600	1267
59	600	1392

The platoon leaves the upstream intersection beginning at t = 30, with a flow rate of 1900 veh/hr. The platoon first reaches the downstream intersection at t = 54 sec (since t' = 23.4 sec, or nearly 24 sec), with an arrival flow rate of 355 veh/hr as shown in the calculation below.

$$q_{54} = Fq_{30} + (1 - F)q_{53}$$

$$q_{54} = .197(1900) + .803(0) = 375\ veh/hr$$

Figure 74 shows the resulting flow profiles, the lower flow profile shows the departure pattern from the upstream intersection and the upper flow profile the arrival pattern at the downstream intersection.

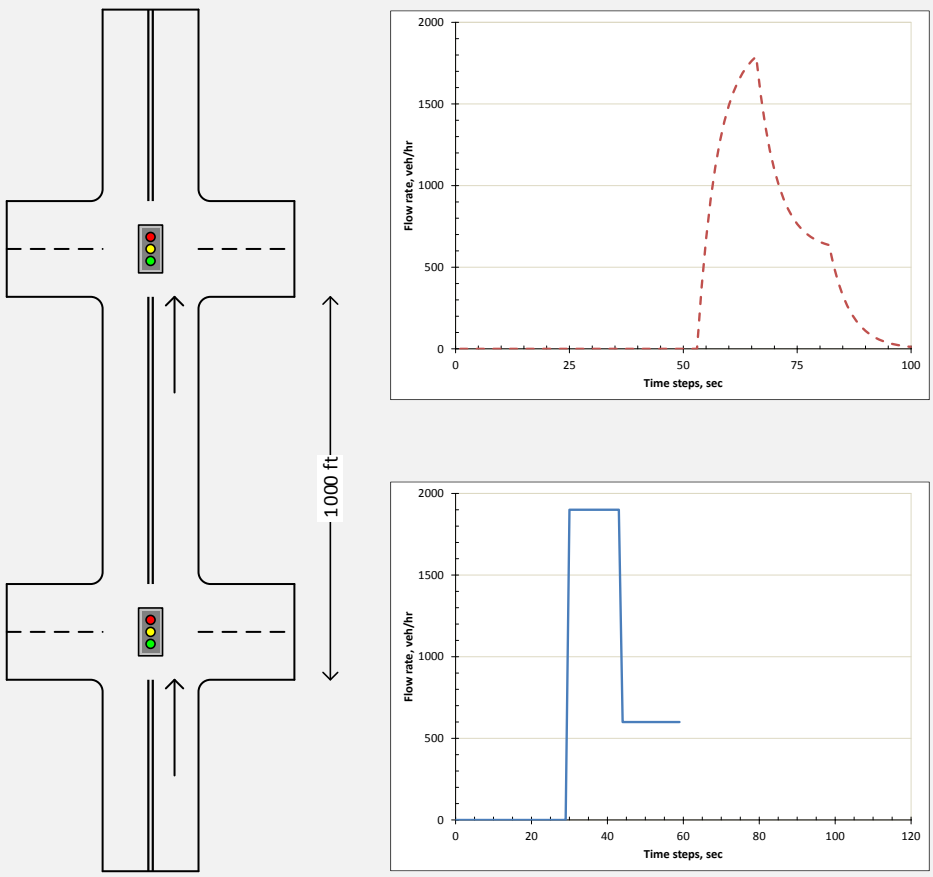


Figure 74. Intersection layout and flow profile diagrams for Example Calculation 18

If the offset for the downstream intersection is set to zero, that is the green interval for the EBTH movements begin at the same time for both intersections, the queue accumulation polygon for the downstream intersection shown in Figure 75 results. The queue accumulation polygon shows that while the first part of the platoon arrives at t = 54 sec (see Table 8), the majority of the platoon arrives after the beginning of red, at t = 60 sec. The queue grows to a maximum value of 7.8 veh at t = 89 sec, and begins to dissipate thereafter. The total delay, the area of the queue accumulation polygon, is the sum of the value of the queue length at each one second time step. The result is 211.7 veh-sec of delay. The average delay, the total delay divided by the total number of vehicles that are served during the cycle, is 21.2 sec/veh.

$$d = \frac{211.2\ sec}{10\ veh/cycle} = 21.2\ sec/veh$$

We can compare this result with the average delay calculated using Equation 25, which assumes uniform arrivals.

$$d_{avg} = \frac{(0.5r)(1 - g/C)}{(1 - v/s)} = \frac{(0.5)(30)(0.5)}{1 - (600/1900)} = \frac{7.5}{0.6842} = 11.0\ sec/veh$$

In this case, when more than three-quarters of the vehicles arrive during red, the delay is more than double than for the case when the arrival pattern is uniformly during red and green.

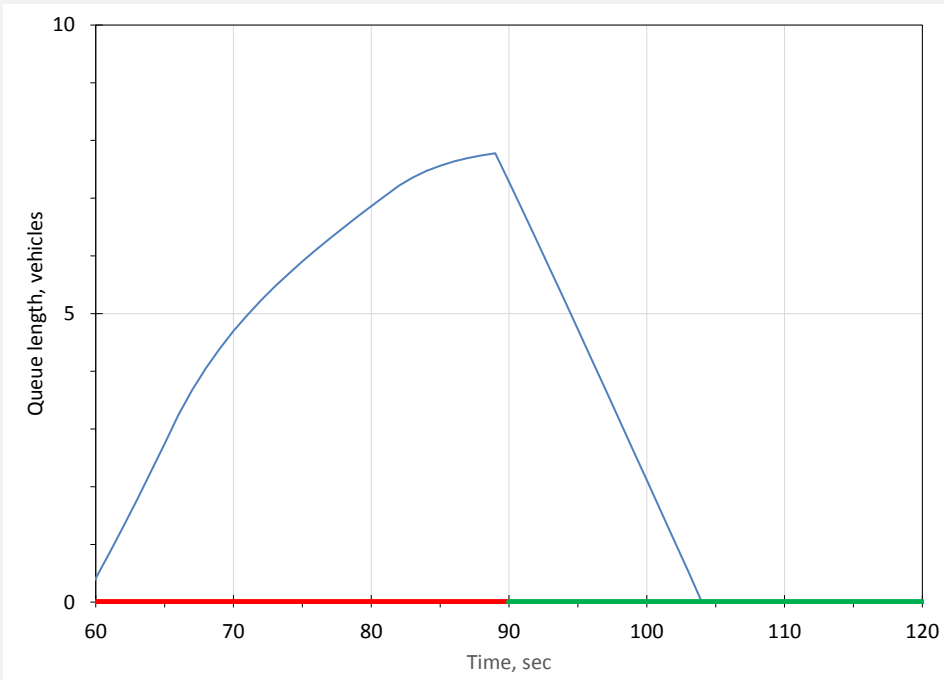


Figure 75. Arrival flow profile at downstream intersection, Example Calculation 18

12. Scenario 7. Predicted Average Green Time under Actuated Control

What is the signalized intersection is operating under actuated control? The green time needed to calculate the approach capacity varies from cycle to cycle, based on the demand at each intersection approach. The higher the demand, the longer the green time is extended, subject to the limitation of the maximum green time.

In Scenario 7, we consider the prediction of the average green time under actuated signal control. The scenario is based on the intersection of two one-way streets, each with one lane and with through movements only, as illustrated in Figure 76. The green time prediction model for actuated signal control is based on work by Akcelik (x), Courage (x), and modified for the HCM 2010 by Bonneson (x).

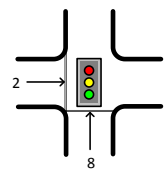
Pretimed ☐ Actuated ☒	
Demand < capacity	
TH only	
Uniform arrivals	

Figure 76. Scenario 7

Relationship between Timing and Interval Parameters

Figure 77 shows the relationship between the two variables predicted by the model (the queue service time and the green extension time) and the effective green and red times, the displayed green and red times, and the lost times using a queue accumulation polygon. The queue service time (g_s) is the time that it takes for the queue to clear after the beginning of the green interval and the green extension time (g_e) is the time that the green is likely to extend after the queue has cleared. Two variables constrain these predictions, the minimum green time and the maximum green time. For actuated signal control the displayed green must be at least equal to the minimum green time (G_{min}) but less than the maximum green time (G_{max}).

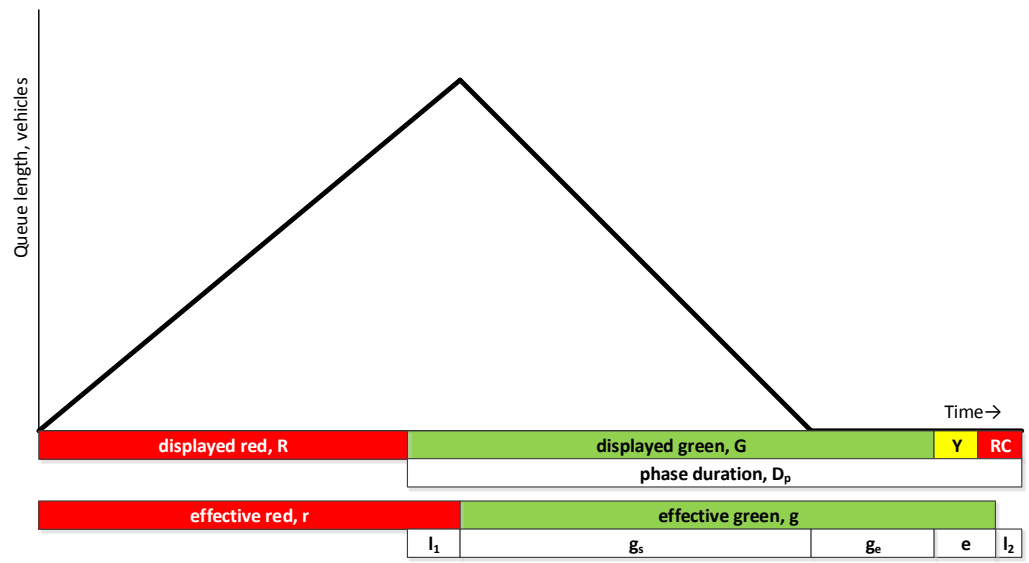


Figure 77. Green time intervals, actuated control model

Queue Service Time

In section 8 of this chapter, the queue service time was defined as the length of the queue at the end of the effective red (vr) divided by the rate of queue clearance after the start of effective green (s-v).

Equation 33

$$g_s = \frac{vr}{s - v}$$

where

g_s = queue service time, sec
v = arrival flow rate, veh/hr (or veh/sec)
r = effective red, sec
s = saturation flow rate, veh/hr or veh/sec

The green time prediction model assumes that the actuated controller will continue to keep the phase active (and green will continue to be displayed) at least as long as this queue is being served.

The model also considers non-uniform arrivals, as described in section 11 of this chapter, with one flow rate during red and another during green. Equation 33 can be modified to consider these two arrival rates as shown in Equation 34.

Equation 34

$$g_s = \frac{v_r r}{s - v_g}$$

where

v_r = arrival rate during red, and
v_g = arrival rate during green.

Since we can predict the proportion of vehicles that arrive during green (P) using the method from section 11, we can develop a form for Equation 34 based on P, not directly on the arrival flow rates during red and green.

To do this, we note that:

Equation 35

$$P = \frac{vehicle\ count\ during\ green}{vehicle\ count\ during\ the\ cycle}$$

We also note that the flow rate during green (v_g) is equal to the vehicle count during green divided by the effective green time, g.

Equation 36

$$v_g = \frac{vehicle\ count\ during\ green}{g}$$

Similarly, the flow rate over the cycle is:

Equation 37

$$v = \frac{vehicle\ count\ during\ cycle}{C}$$

We can then write P using Equation 36 and Equation 37.

Equation 38

$$P = \frac{gv_g}{Cv}$$

Solving for the flow rate during green, we get:

Equation 39

$$v_g = \frac{PCv}{g}$$

We also note that the proportion of vehicles arriving during red can be written as:

Equation 40

$$1 - P = \left(\frac{r}{C}\right)\left(\frac{v_r}{v}\right)$$

Solving for the flow rate during red, v_r:

Equation 41

$$v_r = \frac{(1 - P)(vC)}{r}$$

We can now write the queue service time in terms of results from the predicted arrival flow from section 11: the proportion of vehicles arriving during green (P) and the average flow rate for the cycle (v):

Equation 42

$$g_s = \frac{v_r r}{s - v_g} = \frac{[(1-P)(vC)/r](r)}{s - PCv/g} = \frac{(1-P)(vC)}{s - PCv/g}$$

Example Calculation 19. Calculating the Queue Service Time

Suppose the proportion of vehicles, as predicted using the method of section 11 is 0.75. You are also given that:

- v = 700 veh/hr (q = 0.1944 veh/sec)
- s = 1900 veh/hr = 0.5278 veh/sec
- g = 25 sec
- r = 35 sec
- C = 60 sec

What is the queue service time for these conditions? Using Equation 42 to calculate the queue service time:

$$g_s = \frac{(1-P)(vC)}{s - (PCv/g)} = \frac{(1-0.75)(0.1944)(60)}{0.5278 - (0.75)(60)(0.1944)/25} = \frac{2.916}{0.1779} = 16.4\ sec$$

To check this calculation, as well as to confirm the consistency between Equation 34 and Equation 42:

$$v_g = \frac{PCv}{g} = \frac{(0.75)(60\ sec)(0.1944\ veh/sec)}{25\ sec} = \frac{8.7480}{25} = 0.3499\ veh/\sec = 1260\ veh/hr$$

$$v_r = \frac{(1-P)(vC)}{r} = \frac{(1-0.75)(0.1944\ veh/sec)(60\ sec)}{35\ sec} = \frac{2.916}{35}$$

$$= 0.0833\ veh/\sec = 300\ veh/hr$$

$$g_s = \frac{v_r r}{s - v_g} = \frac{(0.0833\ veh/sec)(35\ sec)}{0.5278\ veh/sec - 0.3499\ veh/sec} = 16.4\ sec$$

Figure 78 shows the results for the calculation of the queue service time, given the values of the cycle length, the effective green time, and the effective red time.

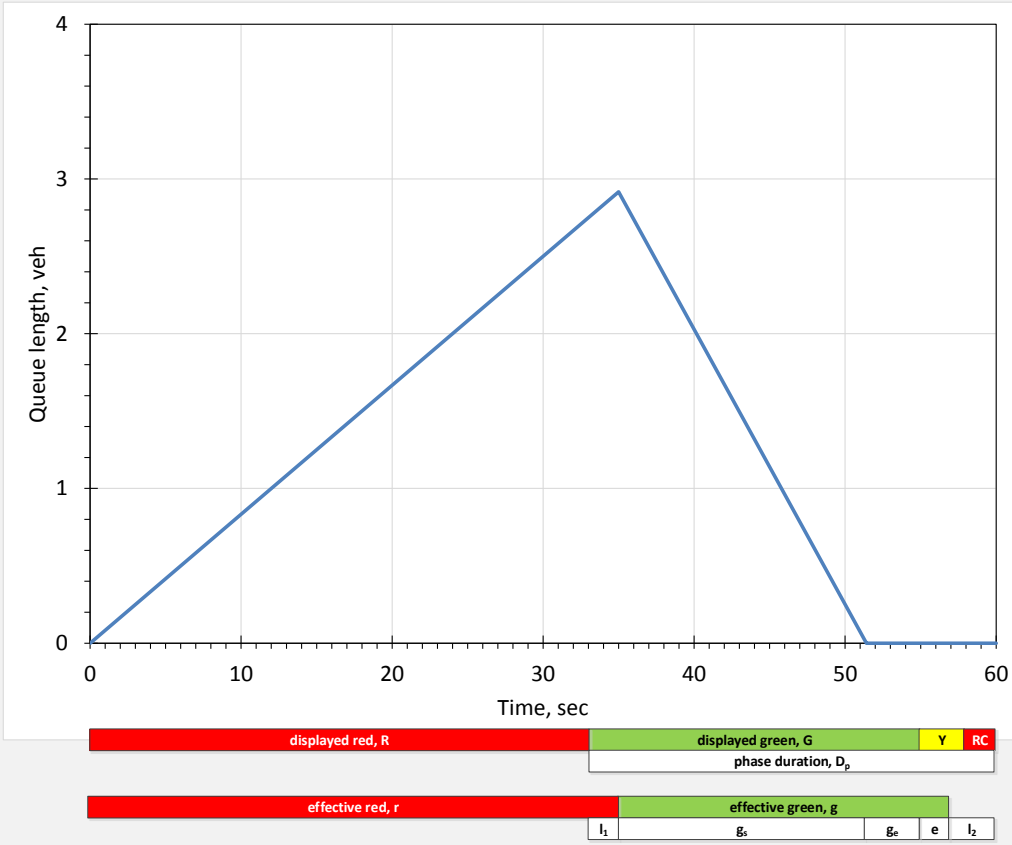


Figure 78. Green intervals, Example Calculation 19

Green Extension Time

Once the minimum green timer has expired, a phase will continue to be active (display green) only as long as the passage timer is active or until the maximum green timer has expired. And, the passage timer will be active as long as the gaps between vehicles are less than the passage time. This process, described in section 4 of this chapter, is shown in more detail here.

Consider the intersection shown in Figure 79. A stop bar detector is located on the approach of a signalized intersection with a length L_d. Vehicle 1 is traveling at a velocity V and has a length L_v. The vehicle enters the detection zone at time 1 and is completely in the zone at time 2. It leaves the zone at time 3. Vehicle 2 enters the zone at time 4.

The time that the detector is occupied by the vehicle, t_o, is given by:

Equation 43

$$t_o = \frac{L_d + L_v}{V}$$

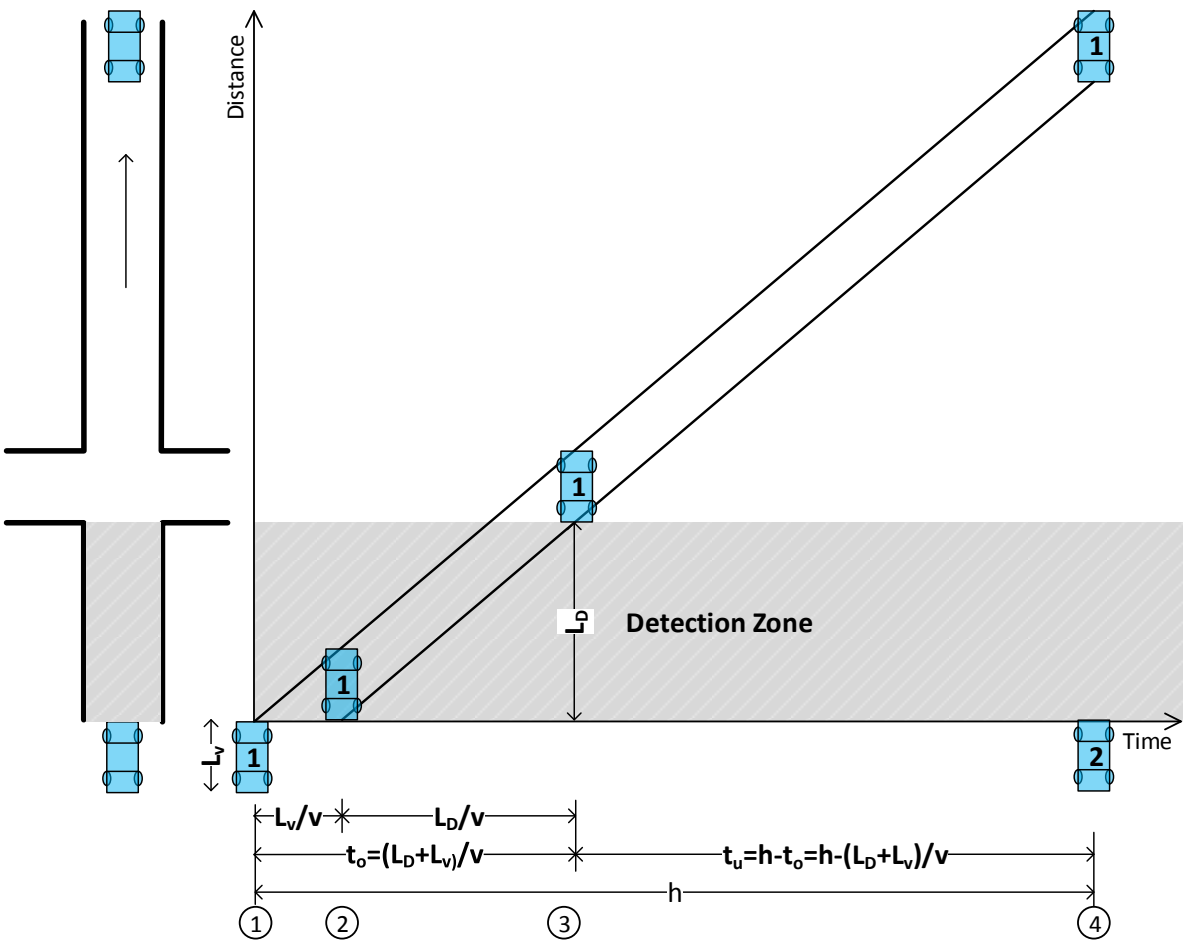


Figure 79. Headway, occupancy time, and unoccupancy time

From the time space diagram in Figure 79, we note that the headway between two vehicles traveling on the intersection approach consists of two parts, the time that the detector is occupied (t_o) and the time that the detector is unoccupied (t_u).

Equation 44

$$h = t_o + t_u$$

Rearranging this equation and substituting the value of t_o from Equation 43, the unoccupancy time t_u can be written:

Equation 45

$$t_u = h - t_o = h - \frac{L_d + L_v}{V}$$

If the maximum time that we will tolerate the detector to be empty (unoccupied) before terminating the phase is t_u, then the passage time PT must be set to this unoccupancy time, t_u. And the headway h becomes the maximum allowable headway between vehicles, MAH, that we are willing to tolerate before green will be terminated. So the passage time can be written:

Equation 46

$$PT = MAH - \frac{L_d + L_v}{V}$$

Solving for MAH, for a given passage time PT:

Equation 47

$$MAH = PT + \frac{L_d + L_v}{V}$$

The prediction of the green extension time is based on the probability that the headways between vehicles arriving after the queue has cleared will be less than the MAH. This prediction in turn is based on the following steps:

- Calculating the number of possible extensions between the serving of the queue and reaching the maximum green time, or max out.
- Calculating the probability that a headway between two vehicles will be less than the MAH

The number of extensions between the serving of the queue and reaching the maximum green time is given by:

Equation 48

$$n = q_g(G_{max} - (g_s - l_1) \geq 0$$

where

q_g = arrival flow rate during green, veh/sec
G_{max} = maximum green time, sec
g_s = queue service time, sec
l_1 = start up lost time, sec

Note that $[G_{max}-(g_s+l_1)]$ is the duration between the ending of the queue service and the maximum green time. Since q_g is the arrival flow rate during green, n is the number of vehicles that would arrive at the intersection before max out, which is simply the number of times that the phase could be extended after the queue has been served.

The probability that a headway between two vehicles arriving after the queue has been served is less than the MAH is given by the cumulative distribution function for a bunched negative exponential distribution.

Equation 49

$$p = 1 - \varphi e^{-\lambda(MAH - \Delta)}$$

where

Δ = headway of bunched vehicles = 1.5 sec for single lane approaches
b = bunching factor = 0.6 for single lane approaches, and,

the proportion of free vehicles is given by:

Equation 50

$$\varphi = e^{-b\Delta q_g}$$

The flow rate parameter is given by:

Equation 51

$$\lambda = \frac{\varphi q_g}{1 - \Delta q_g}$$

where

q_g = arrival flow rate during green.

Bonneson and others [xx] showed that the average number of green extensions (N) before the phase terminates depends on the probability that a headway between vehicles is less than the MAH (p) and the number of extensions from the termination of the queue to the max out point (n).

$$N = \frac{p(1 - p^n)}{1 - p}$$

The green extension time, a function of n and p, is given by:

Equation 52

$$g_e = \frac{N}{q} = \frac{p^2(1 - p^n)}{q(1 - p)}$$

Green Interval Duration

As we noted at the beginning of this section, the duration of the green G is bounded at the lower end by the minimum green time G_{min} and at the upper end by the maximum green time G_{max}.

Equation 53

$$G \geq G_{min}$$

Equation 54

$$G \leq G_{max}$$

Within these constraints, the duration of the green includes three components:

- l_1 = start up lost time, sec
- g_s = green service time, sec
- g_e = green extension time, sec

Considering both the above constraints and the three components listed above, we can write the duration of green as:

Equation 55

$$G = G_{min}, if\ l_1 + g_s + g_e < G_{min}$$

$$G = G_{max}, if\ l_1 + g_s + g_e > G_{max}$$

$$G = l_1 + g_s + g_e, otherwise$$

But we need to consider one more point: what is the probability that the phase will be called? If the arrival flow rate during red is v_r, the probability of the phase being called, p_v, is:

Equation 56

$$p_v = 1 - e^{-q_r C}$$

where:

q_r = arrival flow rate during red, veh/sec,
C = cycle length, sec, and

Equation 57

$$q_r = \frac{v_r}{3600}$$

So, considering the probability of the phase being called as well as Equation 55, the duration of the displayed green is:

Equation 58

$$G = [G_{min}]p_v, if\ l_1 + g_s + g_e < G_{min}$$

$$G = [G_{max}]p_v, if\ l_1 + g_s + g_e > G_{min}$$

$$G = [l_1 + g_s + g_e]p_v, otherwise$$

The duration of the phase D_p is:

Equation 59

$$D_p = G + Y + RC$$

where

Y = yellow interval, sec, and
RC = red clearance interval, sec.

Example Calculation 20. Determining the Effect of Natural Bunching on Flow Rate

Assume a cycle length of 100 sec, g/C = 0.5, Δ = 1.5 sec, and b = 0.6. Let's consider three flow rate examples, as shown in Table 9, 100 veh/hr, 250 veh/hr, and 500 veh/hr. How much does natural bunching of vehicles affect the flow rate parameter?

As the flow rate increases, the proportion of free (non-bunched) vehicles decreases from 0.975 to 0.882. The flow rate parameter, λ, increases in direct proportion to the increase in the flow rate. Figure 80 shows the change in the proportion of free vehicles as a function of the arrival rate, v. Figure 81 shows the relationship between the flow rate parameter λ and the flow rate v.

Table 9. Flow rate data, Example Calculation 20

Flow rate, v veh/hr	Flow rate, q veh/hr	ϕ	λ
100	.0278	.975	.0283
250	.0694	.934	.0728
500	.1389	.883	.1548

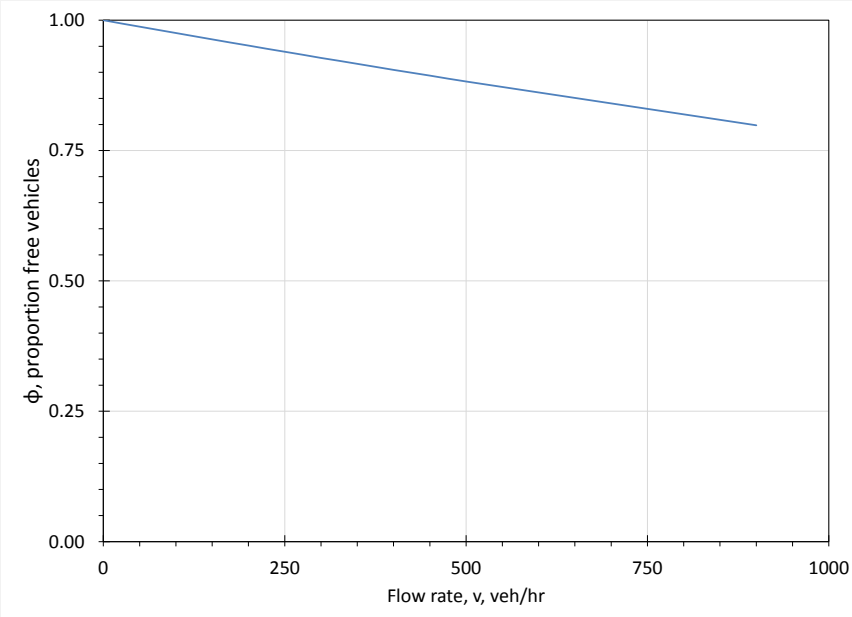


Figure 80. Proportion of free vehicles as a function of flow rate, Example Calculation 20

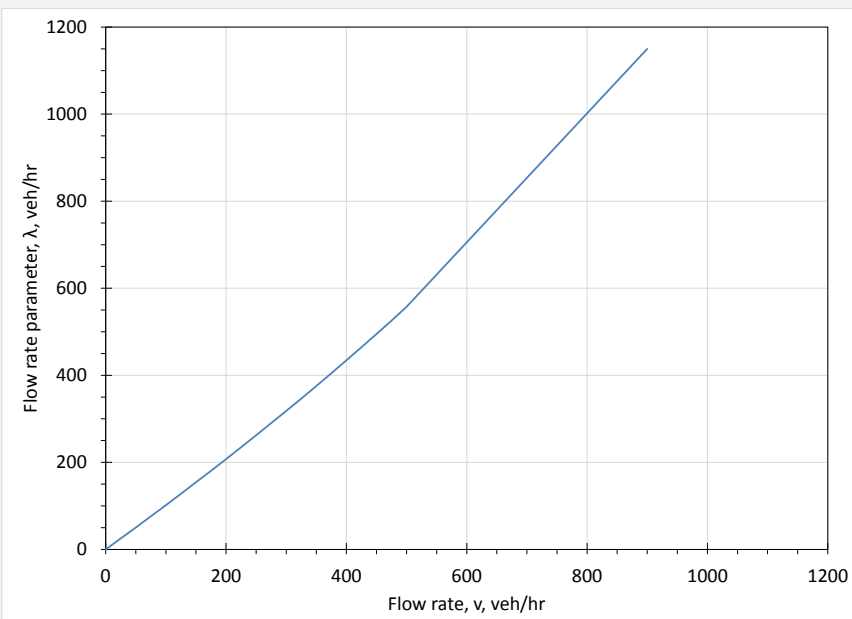


Figure 81. Flow rate parameter as a function of flow rate, Example Calculation 20

Example Calculation 21. Calculating the Green Extension Time under Actuated Control

Consider the following conditions for a signalized intersection under actuated control:

Table 10. Given conditions, Example Calculation 21

Traffic flow conditions	Value
Arrival rate, v	700 veh/hr
Arrival rate, q	0.1944 veh/sec
Saturation flow rate, s	1900 veh/hr
Saturation flow rate, s	0.5278 veh/sec
Proportion vehicles arriving on green, P	0.75

Signal timing conditions	Value
Passage time	2.5 sec
Maximum green time, G_{max}	50 sec
Effective green time, g	30 sec
Cycle length, C	70 sec
Minimum green time, G_{min}	5 sec
Yellow time, Y	3 sec
Red clearance time, RC	2 sec

Other conditions	Value
Detector zone length, L_d	22 ft
Vehicle length, L_v	20vft
Vehicles speed, V	30 mi/hr
Start-up lost time, l_1	2.0 sec

Determine the green extension time based on these conditions.

Step 1. Calculate the MAH. The MAH implied in the given passage time of 2.5 sec is given by Equation 47:

$$MAH = PT + \frac{L_d + L_v}{V} = 2.5\,\text{sec} + \frac{22\,ft + 20\,ft}{\left(30\frac{mi}{hr}\right)(1.47)} = 3.5\,sec$$

Step 2. Calculate queue service time, g_s, using Equation 42.

$$g_s = \frac{(1-P)(vC)}{s-(PCv/g)} = \frac{(1-0.75)(0.1944\,veh/sec)(60\,sec)}{0.5278-(0.75)(60\,sec)(0.1944\,veh/sec)/25\,sec} = \frac{2.916}{0.1779} = 16.4\,sec$$

Step 3. Calculate the number of extensions between serving of the queue and reaching the maximum green time, using Equation 48.

$$n = q_g(G_{max} - (g_s - l_1) = (0.3499)[(50-(16.4+2.0)] = (0.3499)(31.6) = 11.0568$$

Step 4. Calculate the flow rate parameters. The arrival rate during green (from Equation 39):

$$q_g = \frac{PCv}{g} = \frac{(0.75)(60)(.1944)}{25} = 0.3499\,veh/sec$$

The proportion of free vehicles is calculated using Equation 50:

$$\phi = e^{-b\Delta q_g} = e^{-(0.6)(1.5)(0.3499)} = 0.7299$$

The flow rate parameter is calculated using Equation 51:

$$\lambda = \frac{\varphi q}{1 - \Delta q} = \frac{(0.7299)(0.3499\ veh/sec)}{1 - (1.5)(0.3499\ veh/sec)} = \frac{0.2554}{0.4752} = 0.5375\ veh/sec$$

Note that the effect of bunching produces a flow rate parameter (0.5375) that is 54 percent higher than the given arrival flow rate during green of 0.3499 veh/sec.

Step 5. Calculate the probability that a headway between two vehicles arriving after the queue has been served is less than the MAH (Equation 49). This is the same as the probability that green will be extended with the arrival of the next vehicle.

$$p = 1 - \varphi e^{-\lambda(MAH - \Delta)} = 1 - (0.7299)e^{-(0.5375)(3.5-1.5)} = 1 - 0.7299(0.3413) = 0.7509$$

Or, in words, about 75 percent of the time the headway between vehicles is short enough (less than the MAH) for the phase to be extended.

Step 6. Compute the green extension time (Equation 52).

$$g_e = \frac{p^2(1 - p^n)}{q(1 - p)} = \frac{(0.75096^2)(1 - 0.7509^{11.0568})}{(0.3499)(1 - 0.7509)} = \frac{(0.5639)(1 - 0.0421)}{(0.34999)(0.2491)} = 6.1973\ sec$$

This means that green will be extended 6.2 sec beyond the time that the queue has cleared.

Step 7. Calculate the green interval duration (Equation 41).

$$v_r = \frac{(1 - P)(vC)}{r} = \frac{(1 - 0.75)0.1944)(60)}{35} = 0.0833\ veh/sec$$

Using Equation 56:

$$p_v = 1 - e^{-q_r C} = 1 - e^{-(0.0833)(60)} = 0.9932$$

Using Equation 58, the duration of the displayed green is:

$$G = \max(l_1 + g_s + g_e, G_{min})\, p_v = \max(2.0 + 16.4 + 6.2{,}5)\,(0.9932) = 24.4\ sec$$

Using Equation 59, the phase duration is:

$$D_p = G + Y + RC = (24.4 + 3 + 2) = 29.4\ sec$$

---

13. Calculating the Saturation Headway

Saturation Flow Rate

Suppose the signal display has just turned to green, and the vehicles that formed a queue during red begin to move into the intersection. There is some initial delay as the drivers respond to the green display. But soon vehicles are "up to speed" as they enter into and depart from the intersection. As long as the queue that formed during red continues to clear, the flow rate that we would observe at the stop line is called the *saturation flow rate*.

The saturation flow rate was shown earlier in this module (section 2) as part of the representation of traffic flow at a signalized intersection using a D/D/1 queuing model. In the flow profile diagram shown in Figure 27, the departure or service rate during the initial part of green was noted as the saturation flow rate. In Figure 30 (the cumulative vehicle diagram), the slope of the cumulative departure line during the initial portion of green is also equal to the saturation flow rate.

The Highway Capacity Manual suggests a base saturation flow rate of 1900 vehicles per hour per lane be used for traffic analysis. Typically, field measurements show a lower saturation flow rate due to constraining conditions of narrow lanes, on-street parking, the presence of heavy vehicles, and crossing pedestrians. Furthermore, if a left turn movement is permitted (and not protected), its saturation flow rate would be significantly lower since it must yield to opposing through and right turning vehicles.

The headway between vehicles departing at the saturation flow rate is called the *saturation headway*. This parameter was noted in the time-space diagram shown in Figure 24 (section 2 of this module). The relationship between the saturation headway and the saturation flow rate is shown in Equation 60.

Equation 60

$$h_s = \frac{3600\ sec/hr}{s}$$

where

h_s = saturation headway, sec/veh, and

s = saturation flow rate, veh/hr.

Lost Time

Traffic streams are continuously starting and stopping at a signalized intersection as the right-of-way is transferred from one set of traffic movements to another. Each time one set of traffic movements is stopped, change and clearance intervals are provided to allow a time separation before the next set of movements are served. This means that a portion of the cycle cannot be completely utilized, translating to time that is lost to serving traffic. This time is called *lost time*, and includes both time at the beginning of green (start-up lost time) and time at the end of green (clearance lost time).

Startup lost time occurs when the signal indication turns from red to green and drivers in the queue do not instantly start moving at the saturation flow rate; there is an initial lag as drivers react to a change in the signal indication.

The concept of startup lost time is illustrated in Figure 82, which shows the headways between vehicles departing in the queue after the start of green. Typically, the headways for the first four vehicles exceed the saturation headway, as these vehicles react to the change in the signal indication and begin to move into the intersection. The difference between the actual headway and the saturation headway is the lost time for that vehicle. For example, h_1, the headway for the first vehicle in the queue, is the sum of its lost time t_1 and the saturation headway h_s.

The sum of the individual lost times for the first four vehicles is defined as the start-up lost time l_1, as shown in Equation 61.

Equation 61

$$l_1 = \sum_{i=1}^{4} t_i$$

where

t_i = lost time for vehicle i, sec.

Lost time may also occur at the end of a phase. Some traffic may continue into the intersection, even during yellow. But the remainder of the yellow interval and all of the red clearance interval can't be effectively used by traffic and is referred to as *clearance lost time*.

Start-up and clearance lost times are summed to produce the total lost time for a phase as given by Equation 62.

Equation 62

$$t_L = l_1 + l_2$$

where

t_L = total lost time for a phase, sec,

l_1 = start-up lost time, sec, and

l_2 = clearance lost time, sec.

The start-up lost time typically has a value of 2 sec, while the clearance lost time also has a typical value of 2 sec. Thus a value of 4 sec is often used for the total lost time per phase.
As we will see in the next section, the lost time is important in determining how much green time is effectively available to serve traffic demand.

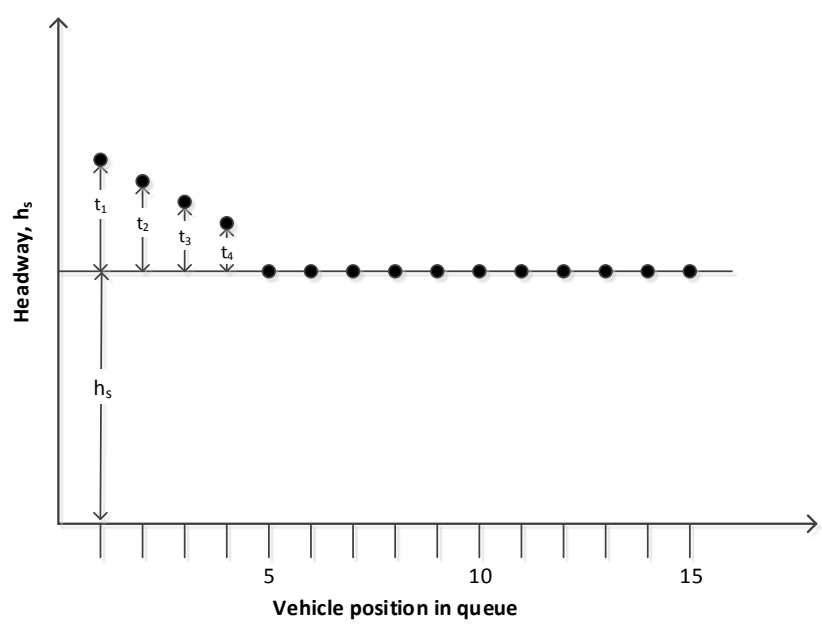


Figure 82. Saturation headway and start-up lost time

Example Calculation 22. Calculating the Start-up Lost Time

Given the following data that have been collected on one approach at a signalized intersection as shown in Table 11. Determine the saturation headway and saturation flow rate. Also determine the start up lost time.

Table 11. Data for Example Calculation 22

Event	ClockTime	Headway
Green begins	0.0	
Veh 1	2.5	2.5
Veh 2	4.9	2.4
Veh 3	7.1	2.2
Veh 4	9.4	2.3
Veh 5	11.3	1.9
Veh 6	13.2	1.9
Veh 7	15.3	2.1
Veh 8	17.1	1.8
Veh 9	18.2	1.1
Veh 10	20.1	1.9
Veh 11	22.1	2.0

Ignoring the headways for the first four vehicles, saturation headway is calculated to be:

$$h_s = \frac{1.9 + 1.9 + 2.1 + 1.8 + 1.1 + 1.9 + 2.0}{7} = \frac{12.7}{7} = 1.81\ sec$$

The start-up lost time is calculated using the headways for the first four vehicles and the saturation headway calculated above.

$$l_1 = (2.5 - 1.81) + (2.4 - 1.81) + (2.2 - 1.81) + (2.3 - 1.81) = 2.14\ sec$$

14. Building the Computational Engines and Exploring the Models

Several of the scenarios that we have developed are complex and often difficult to apply or study without the aid of a computational engine. We will now build four computational engines to allow you to see what these models predict under a variety of traffic flow and signal control conditions. Building the models also gives you a deeper understanding of the relationships between the variables that constitute the models.

- Scenario 2. Calculating the capacity utilization of an intersection using critical movement analysis
- Scenario 3. Calculating uniform delay of an approach when demand is less than capacity.
- Scenario 6. Calculating delay when the arrival pattern is non-uniform.
- Scenario 7. Predicting average green time under actuated control.

Scenario 2. Calculating the Capacity Utilization of an Intersection Using Critical Movement Analysis

In Scenario 2, we calculate the capacity utilization of a signalized intersection using the critical movement analysis method. For this scenario, we consider a four-leg intersection. Each approach has two lanes, an exclusive left turn lane and a through lane. A pretimed signal controls the intersection.

Requirements

The computational engine for Scenario 2 will satisfy the following requirements:

- Accepts volumes for each of the eight movements as inputs.
- Accepts the left turn phasing, the cycle length, the lost time per phase, and the saturation flow rates as inputs.
- Determines the flow ratios for each movement and the flow ratio sums for each concurrency group.
- Determines the critical movements for each concurrency group.
- Determines the critical volume-to-capacity ratio and the sufficiency of capacity for the intersection.

Template

Figure 83 through Figure 85 shows the computational engine template for Scenario 2. The input data are entered in rows 5 through 14 as shown in Figure 83.

	A	B	C	D	E	F	G	H	I	J
1	**CMA computational engine - scenario 2**									
2										
3	**Step 0: Given Data**		**East-West Concurrency Group**				**North-South Concurrency Group**			
4	Volume data	Ring 1	v_1		v_2		v_3		v_4	
5		Ring 2	v_5		v_6		v_7		v_8	
6										
7	LT phasing, EW concurrency group									
8	LT phasing, NS concurrency group									
9	Cycle length		sec							
10	Lost time per phase		sec							
11	Saturation flow rate, TH		veh/hr							
12	Saturation flow rate, protected LT		veh/hr							
13	Saturation flow rate, permitted LT		veh/hr							

Figure 83. Computational engine template, Scenario 2

Figure 84 shows the calculations for steps 1, 2, and 3, in which the critical movements are determined.

Step 1: Compute the flow ratio Y for each movement present at the intersection

	East-West Concurrency Group		North-South Concurrency Group	
Ring 1	v_1	v_2	v_3	v_4
	s_1	s_2	s_3	s_4
	Y_1	Y_2	Y_3	Y_4
Ring 2	v_5	v_6	v_7	v_8
	s_5	s_6	s_7	s_8
	Y_5	Y_6	Y_7	Y_8

Step 2: Determine the flow ratio sums for the phase sequences in each ring for each concurrency group (for the case of permitted LTs only)

	East-West Concurrency Group		North-South Concurrency Group	
Ring 1	v_1	v_2	v_3	v_4
	s_1	s_2	s_3	s_4
	Y_1	Y_2	Y_3	Y_4
	Y_{EW1}		Y_{NS1}	
Ring 2	v_5	v_6	v_7	v_8
	s_5	s_6	s_7	s_8
	Y_5	Y_6	Y_7	Y_8
	Y_{EW2}		Y_{NS2}	

Step 3: Within each concurrency group, identify the movements with the maximum flow ratio sum (for protected left turns) or the movement with the maximum flow ratio (for permitted left turns). These movements are the critical movements for each concurrency group.

	EW ConGrp	NS ConGrp
Ring 1	v_1	v_3
	s_1	s_3
	Y_1	Y_3
	v_2	v_4
	s_2	s_4
	Y_2	Y_4
Ring 2	v_5	v_7
	s_5	s_7
	Y_5	Y_7
	v_6	v_8
	s_6	s_8
	Y_6	Y_8
	$Y_{EW-critical}$	$Y_{NS-critical}$

	East-West Concurrency Group		North-South Concurrency Group	
Ring 1	v_1	v_2	v_3	v_4
	s_1	s_2	s_3	s_4
	Y_1	Y_2	Y_3	Y_4
	Y_{EW1}		Y_{NS1}	
Ring 2	v_5	v_6	v_7	v_8
	s_5	s_6	s_7	s_8
	Y_5	Y_6	Y_7	Y_8
	Y_{EW2}		Y_{NS2}	
	$Y_{EW-critical}$		$Y_{NS-critical}$	

Figure 84. Computational engine template, Scenario 2

Figure 85 shows steps 4 and 5, in which the critical volume-to-capacity ratio and the sufficiency of capacity are determined for the intersection.

Step 4. Determine the critical volume-to-capacity ratio X_c for the intersection.

$Y_{EW-critical}$	
$Y_{NS-critical}$	
Lost time	
X_c	

Step 5: Based on the value of X_c calculated in step 4, determine the sufficiency of capacity.

Sufficiency of capacity	

Figure 85. Computational engine template, Scenario 2

Example Formulas and VBA Functions

To assist you in constructing the computational engine for Scenario 2, the following figures show some of the required formulas. Figure 86 shows the formulas for calculating the flow ratios for the east-west concurrency group.

	B	C	D	E	F
18		East-West Concurrency Group			
19	Ring 1	v_1	=D5	v_2	=F5
20		s_1	=IF(B8="Permitted",B14,B13)	s_2	=B12
21		Y_1	=D19/D20	Y_2	=F19/F20
22	Ring 2	v_5	=D6	v_6	=F6
23		s_5	=IF(B8="Permitted",B14,B13)	s_6	=B12
24		Y_5	=D22/D23	Y_6	=F22/F23

Figure 86. Example formulas, Scenario 2 computational engine

Figure 87 shows the formulas used to determine the critical movements in the case of protected left turns for the east-west concurrency group for ring 1.

	B	C	D	E	F
27		East-West Concurrency Group			
28	Ring 1	v_1	=IF(B8="Protected",D19,"")	v_2	=IF(B8="Protected",F19,"")
29		s_1	=IF(B8="Protected",D20,"")	s_2	=IF(B8="Protected",F20,"")
30		Y_1	=IF(B8="Protected",D21,"")	Y_2	=IF(B8="Protected",F21,"")
31		Y_{EW1}	=IF(B8="Protected",D30+F30,"")		

Figure 87. Example formulas, Scenario 2 computational engine

Figure 88 shows the formulas that lead to the calculation of X_c.

	A	B	C
55	Step 4. Determine the critical	volume-to-capacity ratio Xc	for the intersection.
56	$Y_{EW\text{-}critical}$	=IF(B8="Protected",K49,D53)	
57	$Y_{NS\text{-}critical}$	=IF(B8="Protected",O49,F53)	
58	Lost time	=LostTime(B8,B9)	
59	X_c	=Xc(B56,B57,B10,B58)	

Figure 88. Example formulas, Scenario 2 computational engine

The computational engine uses two VBA functions, Xc and LostTime. These functions compute the critical volume-to-capacity ratio and the lost time per cycle, respectively. Table 12 shows the VBA code for these functions.

Table 12. VBA functions for Scenario 2 computational engine

```
Public Function Xc(YEW, YNS, Cycle, LostTime)

Xc = (YEW + YNS) * Cycle / (Cycle - LostTime)

End Function
```

```
Public Function LostTime(LTEW, LTNS)

If LTEW = "Protected" Then
    LostTimeEW = 8
ElseIf LTEW = "Permitted" Then
    LostTimeEW = 4
End If

If LTNS = "Protected" Then
    LostTimeNS = 8
ElseIf LTNS = "Permitted" Then
    LostTimeNS = 4
End If

LostTime = LostTimeEW + LostTimeNS

End Function
```

Example Calculation 23

Consider a signalized intersection that with two lanes on each approach (left turn lane and through lane) and protected left turns. Figure 89 shows the given data for this example.

	A	B	C	D	E	F	G	H	I	J
1	**CMA computational engine - scenario 2**									
2										
3	**Step 0: Given Data**		**East-West Concurrency Group**				**North-South Concurrency Group**			
4	Volume data	Ring 1	v_1	125	v_2	600	v_3	175	v_4	550
5		Ring 2	v_5	275	v_6	550	v_7	250	v_8	675
6										
7	LT phasing, EW concurrency group	Protected								
8	LT phasing, NS concurrency group	Protected								
9	Cycle length	90	sec							
10	Lost time per phase	4	sec							
11	Saturation flow rate, TH	1900	veh/hr							
12	Saturation flow rate, protected LT	1900	veh/hr							
13	Saturation flow rate, permitted LT	450	veh/hr							

Figure 89. Example Calculation 23

Figure 90 shows steps 1 and 2 of the critical movement analysis method. The flow ratio sums are calculated for the movements in each ring in each concurrency group and are shown in rows 31 and 35.

	A	B	C	D	E	F	G	H	I	J	K	L
16	**Step 1: Compute the flow ratio Y for each movement present at the intersection**											
17			**East-West Concurrency Group**				**North-South Concurrency Group**					
18		Ring 1	v_1	125	v_2	600	v_3	175	v_4	550		
19			s_1	1900	s_2	1900	s_3	1900	s_4	1900		
20			Y_1	0.066	Y_2	0.316	Y_3	0.092	Y_4	0.289		
21		Ring 2	v_5	275	v_6	550	v_7	250	v_8	675		
22			s_5	1900	s_6	1900	s_7	1900	s_8	1900		
23			Y_5	0.145	Y_6	0.289	Y_7	0.132	Y_8	0.355		
24												
25	**Step 2: Determine the flow ratio sums for the phase sequences in each ring for each concurrency group (for the case of protected LTs only)**											
26			**East-West Concurrency Group**				**North-South Concurrency Group**					
27		Ring 1	v_1	125	v_2	600	v_3	175	v_4	550		
28			s_1	1900	s_2	1900	s_3	1900	s_4	1900		
29			Y_1	0.066	Y_2	0.316	Y_3	0.092	Y_4	0.289		
30			Y_{EW1}		0.382		Y_{NS1}		0.382			
31		Ring 2	v_5	275	v_6	550	v_7	250	v_8	675		
32			s_5	1900	s_6	1900	s_7	1900	s_8	1900		
33			Y_5	0.145	Y_6	0.289	Y_7	0.132	Y_8	0.355		
34			Y_{EW2}		0.434		Y_{NS2}		0.487			

Figure 90. Example Calculation 23

Figure 91 shows that the movements with the maximum flow ratio sum are in ring 2 for both the east-west and the north-south concurrency groups. These movements are the critical movements for each concurrency group.

	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P
36	**Step 3: Within each concurrency group, identify the movements with the maximum flow ratio sum (for protected left turns)**															
37	**or the movement with the maximum flow ratio (for permitted left turns). These movements are the critical movements**															
38	**for each concurrency group.**															
39			**EW ConGrp**		**NS ConGrp**					**East-West Concurrency Group**				**North-South Concurrency Group**		
40		Ring 1	v_1		v_3			Ring 1	v_1	125	v_2	600	v_3	175	v_4	550
41			s_1		s_3				s_1	1900	s_2	1900	s_3	1900	s_4	1900
42			Y_1		Y_3				Y_1	0.066	Y_2	0.316	Y_3	0.132	Y_4	0.289
43			v_2		v_4				Y_{EW1}		0.382		Y_{NS1}		0.382	
44			s_2		s_4			Ring 2	v_5	275	v_6	550	v_7	250	v_8	675
45			Y_2		Y_4				s_5	1900	s_6	1900	s_7	1900	s_8	1900
46		Ring 2	v_5		v_7				Y_5	0.144737	Y_6	0.289	Y_7	0.131579	Y_8	0.355
47			s_5		s_7				Y_{EW2}		0.434		Y_{NS2}		0.487	
48			Y_5		Y_7					$Y_{EW-critical}$	0.434			$Y_{NS-critical}$	0.487	
49			v_6		v_8											
50			s_6		s_8											
51			Y_6		Y_8											
52			$Y_{EW-critical}$		$Y_{NS-critical}$											

Figure 91. Example Calculation 23

Figure 92 shows the results of the critical movement analysis. The critical volume-to-capacity ratio is 1.12. This means that, according to Table 6, the demand will exceed the capacity of the intersection.

	A	B	C	D	E	F	G
54	**Step 4. Determine the critical volume-to-capacity ratio X_c for the intersection.**						
55	$Y_{EW-critical}$	0.434					
56	$Y_{NS-critical}$	0.487					
57	Lost time	16					
58	X_c	1.120					
59							
60	**Step 5: Based on the value of X_c calculated in step 4, determine the sufficiency of capacity.**						
61	Sufficiency of capacity	The demand exceeds available capacity of the intersection					

Figure 92. Example Calculation 23

Example Calculation 24

Consider a signalized intersection that with two lanes on each approach (left turn lane and through lane) with permitted left turns. Figure 93 shows the given data for this example.

	A	B	C	D	E	F	G	H	I	J
3	**Step 0: Given Data**		**East-West Concurrency Group**				**North-South Concurrency Group**			
4	Volume data	Ring 1	v_1	50	v_2	225	v_3	100	v_4	250
5		Ring 2	v_5	75	v_6	200	v_7	75	v_8	325
6										
7	LT phasing, EW concurrency group	Permitted								
8	LT phasing, NS concurrency group	Permitted								
9	Cycle length	90	sec							
10	Lost time per phase	4	sec							
11	Saturation flow rate, TH	1900	veh/hr							
12	Saturation flow rate, protected LT	1900	veh/hr							
13	Saturation flow rate, permitted LT	450	veh/hr							

Figure 93. Example Calculation 24

Figure 94 shows step 1 of the method where the flow ratios are calculated for each movement.

	A	B	C	D	E	F	G	H	I	J
16	**Step 1: Compute the flow ratio Y for each movement present at the intersection**									
17			**East-West Concurrency Group**				**North-South Concurrency Group**			
18		Ring 1	v_1	50	v_2	225	v_3	100	v_4	250
19			s_1	450	s_2	1900	s_3	450	s_4	1900
20			Y_1	0.111	Y_2	0.118	Y_3	0.222	Y_4	0.132
21		Ring 2	v_5	75	v_6	200	v_7	75	v_8	325
22			s_5	450	s_6	1900	s_7	450	s_8	1900
23			Y_5	0.167	Y_6	0.105	Y_7	0.167	Y_8	0.171

Figure 94. Example Calculation 24

Figure 95 shows how the movements with the maximum flow ratio sum are identified. These movements are the critical movements for each concurrency group. In this example, the critical movements are movements 4 and 7.

Step 3: Within each concurrency group, identify the movements with the maximum flow ratio sum (for protected left turns) or the movement with the maximum flow ratio (for permitted left turns). These movements are the critical movements for each concurrency group.

		EW ConGrp		NS ConGrp	
40	Ring 1	v_1	50	v_3	100
41		s_1	450	s_3	450
42		Y_1	0.111	Y_3	0.222
43		v_2	225	v_4	250
44		s_2	1900	s_4	1900
45		Y_2	0.118	Y_4	0.132
46	Ring 2	v_5	75	v_7	75
47		s_5	450	s_7	450
48		Y_5	0.167	Y_7	0.167
49		v_6	200	v_8	325
50		s_6	1900	s_8	1900
51		Y_6	0.105	Y_8	0.171
52		$Y_{EW\text{-}critical}$	0.167	$Y_{NS\text{-}critical}$	0.222

		East-West Concurrency Group		North-South Concurrency Group	
	Ring 1	v_1	v_2	v_3	v_4
		s_1	s_2	s_3	s_4
		Y_1	Y_2	Y_3	Y_4
		Y_{EW1}		Y_{NS1}	
	Ring 2	v_5	v_6	v_7	v_8
		s_5	s_6	s_7	s_8
		Y_5	Y_6	Y_7	Y_8
		Y_{EW2}		Y_{NS2}	
		$Y_{EW\text{-}critical}$		$Y_{NS\text{-}critical}$	

Figure 95. Example Calculation 24

Figure 96 shows the results. The critical volume-to-capacity ratio is 0.71. This means that, according to Table 6, the intersection is operating under capacity.

	A	B
54	**Step 4. Determine the critical volume-to-capacity ratio X_c for the intersection.**	
55	$Y_{EW\text{-}critical}$	0.167
56	$Y_{NS\text{-}critical}$	0.222
57	Lost time	8
58	X_c	0.427
59		
60	**Step 5: Based on the value of X_c calculated in step 4, determine the sufficiency of capacity.**	
61	Sufficiency of capacity	Intersection is operating under capacity

Figure 96. Example Calculation 24

Scenario 3. Calculating the Uniform Delay of an Approach when Demand is less than Capacity

In Scenario 3, we calculate the uniform delay for one approach of a signalized intersection when demand on the approach is less than its capacity. For this scenario, we consider a one-lane approach with through movements only; a pretimed signal controls the intersection.

Requirements

The computational engine for Scenario 3 will satisfy the following requirements:

- Accepts average arrival rates during the cycle, during red, and during green as inputs.

- Accepts the cycle length, the effective red and green times, and the saturation flow rate as inputs.
- Determines the green and ratio ratios.
- Determines the rate of queue clearance, the maximum queue, the rate of queue formation, the rate of queue clearance, and the queue service time.
- Determines the total delay and the average delay per vehicle using the dimensions of the queue accumulation polygon.
- Plots the queue length over time (the queue accumulation polygon)

Figure 97 shows the queue accumulation polygon and its relevant parameters.

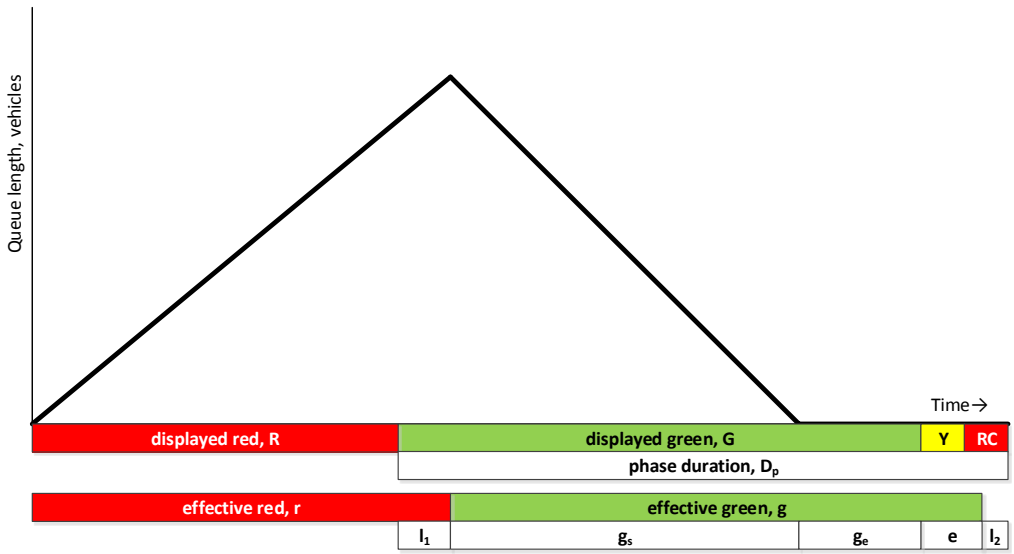


Figure 97. Queue accumulation polygon and relevant parameters, Scenario 3

The following steps are used to determine queue accumulation polygon:

1. Determine the initial queue Qi at the beginning of the effective red.
2. Determine the points in the cycle at which the arrival flow rate or discharge flow rate changes (e.g., when a platoon arrives, when the queue clears)
3. Compute the duration of each time interval identified in step 2.
4. Calculate the capacity of each interval for which there is non-zero discharge.
5. Calculate the v/c ratio.
6. Calculate the queue at the end of each time interval during which there is homogenous flow.
7. Calculate the area of the polygon.
8. Add the areas of each polygon, divide by the number of vehicle arrivals during the cycle, and determine the uniform delay.

Template

Figure 98 shows the computational engine template.

	A	B	C	D	E	F
3	**Given data**	**Value**	**Units**		**Value**	**Units**
4	average arrival rate, v		veh/hr			veh/sec
5	rate during red, v_r		veh/hr			veh/sec
6	rate during green, v_g		veh/hr			veh/sec
7	Cycle length, C		sec			
8	Effective red, r		sec			
9	Effective green, g		sec			
10	Green ratio, g/C					
11	Red ratio, r/C					
12	Saturation flow rate, s		veh/hr			veh/sec
13						
14	**Calculated data - Delay**					
15	Rate of queue clearance		veh/hr			veh/sec
16	Maximum queue (v,r)		veh			
17	Rate of queue formation		veh/hr			veh/sec
18	Rate of queue clearance		veh/hr			veh/sec
19	Queue service time (g_s)		sec			
20	Veh arrive in one cycle		veh			
21	Total delay		veh-sec			
22	Average delay per vehicle		sec/veh			

Figure 98. Computional engine template, Scenario 3

Example Formulas and Visual Basic Code

To assist you in constructing the computational engine for Scenario 3, the following figures show some of the required formulas. Figure 99 shows the formulas for calculating delay as well as the intermediate values required.

	A	B	C	D	E	F
3	**Given data**	**Value**	**Units**		**Value**	**Units**
4	average arrival rate, v	=B5*B11+B6*B10	veh/hr		=B4/3600	veh/sec
5	rate during red, v_r	400	veh/hr		=B5/3600	veh/sec
6	rate during green, v_g	250	veh/hr		=B6/3600	veh/sec
7	Cycle length, C	75	sec			
8	Effective red, r	40	sec			
9	Effective green, g	35	sec			
10	Green ratio, g/C	=B9/B7				
11	Red ratio, r/C	=B8/B7				
12	Saturation flow rate, s	1900	veh/hr		=B12/3600	veh/sec
13						
14	**Calculated data - Delay**					
15	Rate of queue clearance	=B12-B6	veh/hr		=B15/3600	veh/sec
16	Maximum queue (v,r)	=E5*B8	veh			
17	Rate of queue formation	=B5	=C5		=E5	=F5
18	Rate of queue clearance	=B12-B6	veh/hr		=B18/3600	=F6
19	Queue service time (g_s)	=(E5*B8)/(E12-E6)	sec			
20	Veh arrive in one cycle	=E4*B7	veh			
21	Total delay	=0.5*(B8+B19)*B16	veh-sec			
22	Average delay per vehicle	=B21/(E4*B7)	sec/veh			

Figure 99. Example formulas, Scenario 3 computational engine

The computational engine uses two Visual Basic functions, AverageUniformDelay and LOS. These functions compute the average uniform delay and determine the level of service. Table 13 shows the VBA code for these functions.

Table 13. VBA functions used in computational engine for Scenario 3

```
Public Function AverageUniformDelay(r, g, C, s, v)
term1 = 0.5 * r
term2 = 1 - g / C
term3 = 1 - v / s
AverageUniformDelay = (term1 * term2) / term3
End Function
```

```
Public Function LOS(delay)
If delay <= 10 Then
    LOS = "A"
ElseIf delay <= 20 Then
    LOS = "B"
ElseIf delay <= 35 Then
    LOS = "C"
ElseIf delay <= 55 Then
    LOS = "D"
ElseIf delay <= 80 Then
    LOS = "E"
Else
    LOS = "F"
End If
End Function
```

Example Calculation 25

Consider one approach of a signalized intersection, with one lane and through movements only. Figure 100 shows the given data for this example.

	A	B	C	D	E	F
3	**Given data**	**Value**	**Units**		**Value**	**Units**
4	average arrival rate, v	330	veh/hr		0.092	veh/sec
5	rate during red, v_r	400	veh/hr		0.111	veh/sec
6	rate during green, v_g	250	veh/hr		0.069	veh/sec
7	C	75	sec			
8	r	40	sec			
9	g	35	sec			
10	g/C	0.467				
11	r/C	0.533				
12	s	1900	veh/hr		0.528	veh/sec

Figure 100. Example Calculation 25

Figure 101 shows the delay calculation results. The maximum queue, which occurs at the end of red, is 4.4 vehicles. The rate of queue formation is the arrival rate during red, 400 veh/hr. The rate of queue clearance is the saturation flow rate minus the arrival rate during green. The average delay per vehicle, shown in cell B22, is 16.1 sec/veh.

	A	B	C	D	E	F
14	**Calculated data - Delay**					
15	Rate of queue clearance	1650	veh/hr		0.458	veh/sec
16	Maximum queue ($v_r r$)	4.4	veh			
17	Rate of queue formation	400	veh/hr		0.111	veh/sec
18	Rate of queue clearance	1650	veh/hr		0.458	veh/sec
19	Queue service time (g_s)	9.7	sec			
20	Veh arrive in one cycle	6.9	veh			
21	Total delay	110.4	veh-sec			
22	Average delay per vehicle	16.1	sec/veh			

Figure 101. Example Calculation 25

Figure 102 shows the calculations to plot the queue accumulation polygon as well as the resulting polygon.

	I	J	K	L	M	N	O	P	Q	R
3	**Queue Accumulation Polygon**									
4	**Required data**	**Queue**	**Time**		**Timing intervals**	**Y**	**X**			
5	Origin	0.0	0.0		Begin red	0	0			
6	End of red	4.4	40.0		End red	0	40			
7	Queue clears	0.0	49.7		Begin green	0	40			
8	End of green	0.0	75.0		End green	0	75			

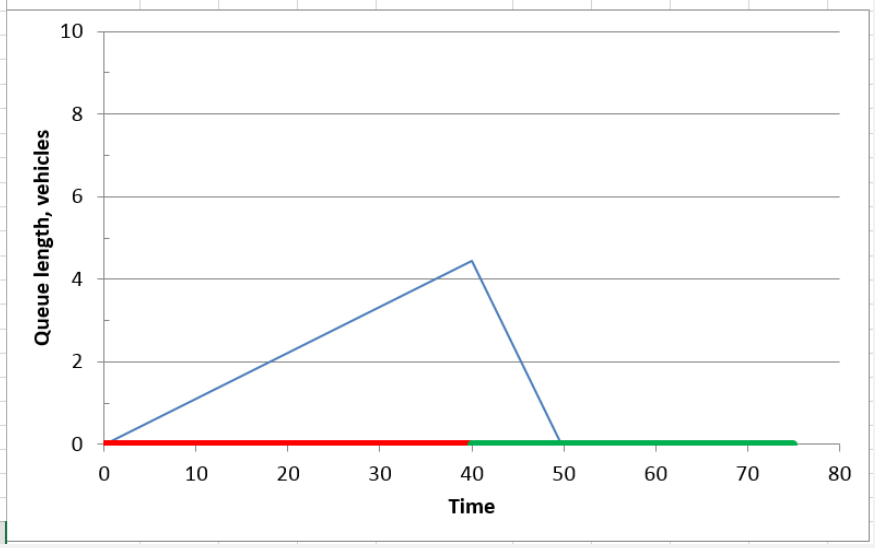


Figure 102. Example Calculation 25

Scenario 6. Calculating Delay when the Arrival Pattern is Non-Uniform

In Scenario 6, we calculate the delay for one approach of a signalized intersection when the arrival pattern is determined by the flows from an upstream traffic signal. The approach has one lane, with through movements only. A pretimed signal controls both intersections.

Requirements

The computational engine for Scenario 6 will satisfy the following requirements:

- Accepts cycle length, green time, red time, saturation flow rate and arrival rate for the upstream signalized intersection as inputs.
- Accepts the offset, the cycle length, the green time, and the red time of the downstream intersection as inputs.
- Accepts distance between the two intersections and the vehicle speed as inputs.
- Calculates the queue service time at the upstream intersection, and the platoon dispersion model parameters.
- Calculates the departure flow profile at the upstream intersection and the arrival flow profile at the downstream intersection.
- Calculates the queue status during each time step at the downstream intersection (how much the queue changes during the time step) and the queue length at the end of each time step.
- Plots the flow profile patterns (the departure profile at the upstream intersection and the arrival profile at the downstream intersection) and the resulting queue accumulation polygon at the downstream intersection.

- Calculates the total delay and the average delay at the downstream intersection.

Template

Figure 103 and Figure 104 show the computational engine template.

	A	B	C	D	E	F	G	H	I	J	K
3	**Upstream Signal**				**Platoon Dispersion Model**				**Downstream Signal**		
4	Cycle length, C		sec		Speed		mi/hr		Distance		ft
5	Effective green, g		sec		t_R		sec		Offset		sec
6	Effective red, r		sec		tprime		sec		Cycle length, C		sec
7	Saturation flow rate, s		veh/hr-g		F				Effective green, g		sec
8	ArrFlow		veh/hr						Effective red, r		sec
9	Queue service time, g_s		sec						Queue service time, g_s		sec

Figure 103. Computational engine template, Scenario 6

In Figure 104, the following variables are used:

- Time i = time step i, sec
- DepFlowi = departure flow rate, veh/hr, from the upstream intersection during time step i.
- ArrFlowj (column E) = arrival flow rate, veh/hr, at the downstream intersection during time step j.
- ArrFlowj (column F) = arrival flow rate, veh/sec, at the downstream intersection during time step j.
- QueueStatus = the increase or decrease in the queue, in vehicles, during time step j.
- Queue = length of the queue, in vehicles, at the end of time step j.

	A	B	C	D	E	F	G	H
12	**Flow Profile Departing**			**Flow Profile Arriving**				
13	**Time i**	**DepFlowi**		**Time i**	**ArrFlowj**	**ArrFlowj**	**QueueStatus**	**Queue**
14								
15								
16								
17								
18								
19								
20								
21								
22								
23								
24								
25								
26								
27								
28								
29								
30								
31								
32								
33								

Figure 104. Computational engine template, Scenario 6

Example Formulas and Visual Basic functions

To assist you in constructing the computational engine for Scenario 6, the following figures show some of the required formulas. Figure 105 shows the formulas for calculating the flow rate departing from the upstream intersection for time steps 0 through 10. This calculation uses the function DepFlowi, defined in Table 14.

The inputs for this function include the following parameters that apply to the upstream intersection:

- Queue service time ($B$9)
- Effective green time ($B$5)
- Cycle length ($B$4)
- Arrival flow rate ($B$8)
- Saturation flow rate ($B$7)
- Time step (A14…A24)

	A	B
12		**Flow Profile Departing**
13	**Time i**	**DepFlowi**
14	0	=DepFlowi($B$9,$B$5,$B$4,$B$8,$B$7,A14)
15	1	=DepFlowi($B$9,$B$5,$B$4,$B$8,$B$7,A15)
16	2	=DepFlowi($B$9,$B$5,$B$4,$B$8,$B$7,A16)
17	3	=DepFlowi($B$9,$B$5,$B$4,$B$8,$B$7,A17)
18	4	=DepFlowi($B$9,$B$5,$B$4,$B$8,$B$7,A18)
19	5	=DepFlowi($B$9,$B$5,$B$4,$B$8,$B$7,A19)
20	6	=DepFlowi($B$9,$B$5,$B$4,$B$8,$B$7,A20)
21	7	=DepFlowi($B$9,$B$5,$B$4,$B$8,$B$7,A21)
22	8	=DepFlowi($B$9,$B$5,$B$4,$B$8,$B$7,A22)
23	9	=DepFlowi($B$9,$B$5,$B$4,$B$8,$B$7,A23)
24	10	=DepFlowi($B$9,$B$5,$B$4,$B$8,$B$7,A24)

Figure 105. Example formulas, Scenario 6 computation engine

Figure 106 shows the formulas for calculating the flow rate arriving at the downstream intersection for time steps 0 through 10 in column E. This calculation uses the function ArrFlow_j, defined in Table 14, to calculate the arrival flow during time step j. The inputs for this function include the following parameters.

- Departure flow from the upstream intersection at time step i.
- Arrival flow at the downstream intersection at time step j-1.
- F, the smoothing parameter defined in Equation 31.

The function also uses the Excel function "INDIRECT', which determines a reference to a range, in this case the reference to the cell with the arrival flow rate, i-j time steps previously. The arrival flow calculated in column F is the rate in veh/sec based on the result (in veh/hr) in column E.

	D	E	F
12		**Flow Profile Arriving**	
13	**Time i**	**ArrFlowj**	**ArrFlowj**
14	0	=IF(A14>$F$6+$B$6,ArrFlow_j(INDIRECT("b"&ROW()-INT($F$6)),E13,$F$7),0)	=E14/3600
15	1	=IF(A15>$F$6+$B$6,ArrFlow_j(INDIRECT("b"&ROW()-INT($F$6)),E14,$F$7),0)	=E15/3600
16	2	=IF(A16>$F$6+$B$6,ArrFlow_j(INDIRECT("b"&ROW()-INT($F$6)),E15,$F$7),0)	=E16/3600
17	3	=IF(A17>$F$6+$B$6,ArrFlow_j(INDIRECT("b"&ROW()-INT($F$6)),E16,$F$7),0)	=E17/3600
18	4	=IF(A18>$F$6+$B$6,ArrFlow_j(INDIRECT("b"&ROW()-INT($F$6)),E17,$F$7),0)	=E18/3600
19	5	=IF(A19>$F$6+$B$6,ArrFlow_j(INDIRECT("b"&ROW()-INT($F$6)),E18,$F$7),0)	=E19/3600
20	6	=IF(A20>$F$6+$B$6,ArrFlow_j(INDIRECT("b"&ROW()-INT($F$6)),E19,$F$7),0)	=E20/3600
21	7	=IF(A21>$F$6+$B$6,ArrFlow_j(INDIRECT("b"&ROW()-INT($F$6)),E20,$F$7),0)	=E21/3600
22	8	=IF(A22>$F$6+$B$6,ArrFlow_j(INDIRECT("b"&ROW()-INT($F$6)),E21,$F$7),0)	=E22/3600
23	9	=IF(A23>$F$6+$B$6,ArrFlow_j(INDIRECT("b"&ROW()-INT($F$6)),E22,$F$7),0)	=E23/3600
24	10	=IF(A24>$F$6+$B$6,ArrFlow_j(INDIRECT("b"&ROW()-INT($F$6)),E23,$F$7),0)	=E24/3600

Figure 106. Example formulas, Scenario 6 computational engine

Figure 107 shows the formulas for calculating the queue status, the increase or decrease in the number of vehicles in the queue during that time step, and the queue, the resulting or current

number of vehicles in the queue at the end of that time step. The formulas, using the function QueueStatus and QueueLength, are shown for time steps 0 through 10.

	G	H
12		
13	**QueueStatus**	**Queue**
14	=QueueStatus(D14,F14,$N$10,$N$11,$J$9,$N$13,H13)	
15	=QueueStatus(D15,F15,$N$10,$N$11,$J$9,$N$13,H14)	=QueueLength(D15,$N$10,$N$11,H14,G15,$J$9)
16	=QueueStatus(D16,F16,$N$10,$N$11,$J$9,$N$13,H15)	=QueueLength(D16,$N$10,$N$11,H15,G16,$J$9)
17	=QueueStatus(D17,F17,$N$10,$N$11,$J$9,$N$13,H16)	=QueueLength(D17,$N$10,$N$11,H16,G17,$J$9)
18	=QueueStatus(D18,F18,$N$10,$N$11,$J$9,$N$13,H17)	=QueueLength(D18,$N$10,$N$11,H17,G18,$J$9)
19	=QueueStatus(D19,F19,$N$10,$N$11,$J$9,$N$13,H18)	=QueueLength(D19,$N$10,$N$11,H18,G19,$J$9)
20	=QueueStatus(D20,F20,$N$10,$N$11,$J$9,$N$13,H19)	=QueueLength(D20,$N$10,$N$11,H19,G20,$J$9)
21	=QueueStatus(D21,F21,$N$10,$N$11,$J$9,$N$13,H20)	=QueueLength(D21,$N$10,$N$11,H20,G21,$J$9)
22	=QueueStatus(D22,F22,$N$10,$N$11,$J$9,$N$13,H21)	=QueueLength(D22,$N$10,$N$11,H21,G22,$J$9)
23	=QueueStatus(D23,F23,$N$10,$N$11,$J$9,$N$13,H22)	=QueueLength(D23,$N$10,$N$11,H22,G23,$J$9)
24	=QueueStatus(D24,F24,$N$10,$N$11,$J$9,$N$13,H23)	=QueueLength(D24,$N$10,$N$11,H23,G24,$J$9)

Figure 107. Example formulas, Scenario 6 computational engine

The computational engine uses six Visual Basic functions. Table 14 lists each of these functions and its purpose.

Table 14. VBA functions, Scenario 6 computational engine

Function	Purpose
F	This function calculates the smoothing function F defined in Equation 31. The function requires one input: • tR, the running time between the upstream and downstream intersections
ArrFlow_j	This function calculates the flow rate arriving at the downstream intersection at time step j, as defined in Equation 30. The function requires the following inputs: • DepFlowi, the departing flow rate from the upstream intersection at time step i • ArrFlow_jminus1, the arrival flow rate at the downstream intersection at time step j-1 • F, the smoothing function
tprime	This function calculates the travel time for the front of the platoon from the upstream intersection to the downstream intersection as defined in Equation 32. The function requires the following inputs: • tR, the running time between the upstream and downstream intersections • F, the smoothing function
DepFlowi	This function calculates the flow rate departing from the upstream intersection during time step i. The function requires the following inputs: • Queue service time • Effective green time • Cycle length • Arrival flow rate • Saturation flow rate • Current time step
QueueStatus	This function calculates the incremental increase or decrease in the queue during the current time step. The function requires the following inputs: • Current time step • Arrival flow rate • Time step for start of red • Time step for end of red • Queue service time • Time step for end of green • Length of queue at the end of the previous time step
QueueLength	This function calculates the length of the queue at the end of the current time step. The function requires the following inputs: • Current time step • Time step for start of red • Time step for end of red • Length of queue at the end of the previous time step • Change in queue length during current time step • Queue service time

Table 15 shows the Visual Basic code for the functions listed in Table 14.

Table 15. Visual Basic code, Scenario 6 computational engine

```
Public Function F(tR)
'Equation 30-21
    F = 1 / (1.315 + 0.138 * tR)
End Function
```

```
Public Function ArrFlow_j(DepFlowi, ArrFlow_jminus1, F)

'Equation 30-19
    ArrFlow_j = F * DepFlowi + (1 - F) * ArrFlow_jminus1
End Function
```

```
Public Function tprime(tR, F)
'Equation 30-22
tprime = tR - (1 / F) + 1.25
End Function
```

```
Public Function DepFlowi(gq, G, C, v, SatFlow, time)
R = C - G

If time < R Then
    DepFlowi = 0
ElseIf time < G + gq Then
    DepFlowi = SatFlow
ElseIf time < C Then
```

```
    DepFlowi = v
ElseIf time < C + R Then
    DepFlowi = 0
ElseIf time < C + R + gq Then
    DepFlowi = SatFlow
ElseIf time < 2 * C Then
    DepFlowi = v
ElseIf time < 2 * C + R Then
    DepFlowi = 0
ElseIf time < 2 * C + R + gq Then
    DepFlowi = SatFlow
ElseIf time < 3 * C Then
    DepFlowi = v
End If

End Function
```

```
Public Function QueueStatus(time, ArrFlow, RedStart2, RedEnd2, gs, GreenEnd3,
QueueLengthPrevious)

If time < RedStart2 Then
    QueueStatus = 0
ElseIf time < RedEnd2 Then
    QueueStatus = ArrFlow
ElseIf time < RedEnd2 + gs Then
    QueueStatus = ArrFlow - (1900 / 3600)
ElseIf time < GreenEnd2 Then
    QueueStatus = 0
End If

If QueueLengthPrevious = 0 And time > RedStart2 Then
    QueueStatus = 0
End If

End Function
```

```
Public Function QueueLength(time, RedStart2, RedEnd2, QueueLengthPrevious, QueueChange, gs)
If time < RedStart2 Then
    QueueLength = 0
ElseIf time < RedEnd2 + gs Then
    QueueLength = QueueLengthPrevious + QueueChange
ElseIf time >= gs Then
    QueueLength = 0
End If

If QueueLengthPrevious = 0 And time > RedStart2 Then
    QueueLength = 0
End If

If QueueLength < 0 Then
    QueueLength = 0
End If

End Function
```

Example Calculation 26

Consider an arterial with two signalized intersections, with only EB through movements. Figure 108 shows the given data for this example. The offset is set to 30 sec for the downstream intersection.

	A	B	C	D	E	F	G	H	I	J	K
3	**Upstream Signal**				**Platoon Dispersion Model**				**Downstream Signal**		
4	Cycle length, C	60	sec		Speed	25	mi/hr		Distance	1000	ft
5	Effective green, g	30	sec		t_R	27.2	sec		Offset	30	sec
6	Effective red, r	30	sec		tprime	23.4	sec		Cycle length, C	60	sec
7	Saturation flow rate, s	1900	veh/hr-g		F	0.197			Effective green, g	30	sec
8	ArrFlow	800	veh/hr						Effective red, r	30	sec
9	Queue service time, g_s	21.8	sec						Queue service time, g_s	21.8	sec

Figure 108. Given data, Example Calculation 26, offset = 30 sec

Figure 109 shows the departure flow profile and Figure 110 shows the arrival flow profile for this case. Note that the color indications for the green and red are not shown. They will be added later in the example calculation.

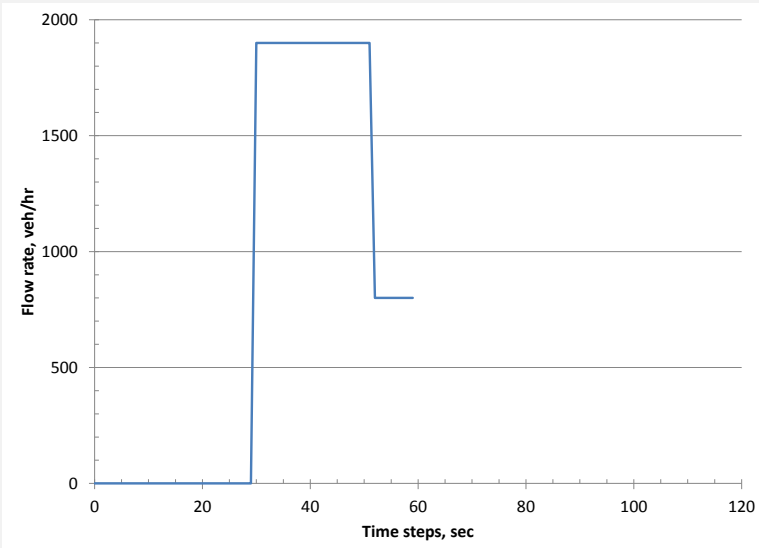


Figure 109. Departure flow profile at upstream intersection, Example Calculation 26, offset = 30 sec

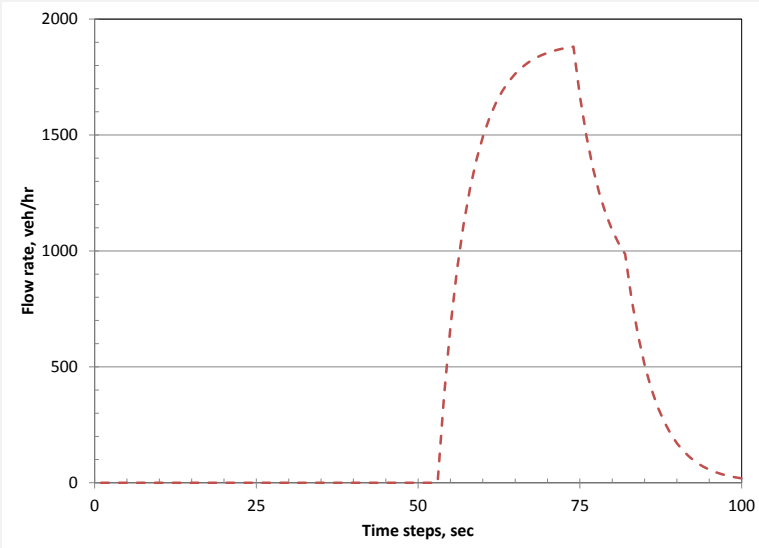


Figure 110. Arrival flow profile at downstream intersection, Example Calculation 26, offset = 30 sec

The start of green at the upstream intersection is at t = 30 sec. Figure 111 shows the departure flow file from t = 30 to t = 53. The departure flow rate is 1900 veh/hr from t = 30 through t = 51, when the queue clears. At t = 52, the departure flow rate equals the arrival flow rate of 800 veh/hr. There are no data for the arrival flow profile since the platoon from the downstream intersection has not yet arrived.

	A	B	C	D	E	F	G	H
12	Flow Profile Departing			Flow Profile Arriving				
13	Time i	DepFlowi		Time i	ArrFlowj	ArrFlowj	QueueStatus	Queue
44	30	1900		30	0	0.000	0.00	0.00
45	31	1900		31	0	0.000	0.00	0.00
46	32	1900		32	0	0.000	0.00	0.00
47	33	1900		33	0	0.000	0.00	0.00
48	34	1900		34	0	0.000	0.00	0.00
49	35	1900		35	0	0.000	0.00	0.00
50	36	1900		36	0	0.000	0.00	0.00
51	37	1900		37	0	0.000	0.00	0.00
52	38	1900		38	0	0.000	0.00	0.00
53	39	1900		39	0	0.000	0.00	0.00
54	40	1900		40	0	0.000	0.00	0.00
55	41	1900		41	0	0.000	0.00	0.00
56	42	1900		42	0	0.000	0.00	0.00
57	43	1900		43	0	0.000	0.00	0.00
58	44	1900		44	0	0.000	0.00	0.00
59	45	1900		45	0	0.000	0.00	0.00
60	46	1900		46	0	0.000	0.00	0.00
61	47	1900		47	0	0.000	0.00	0.00
62	48	1900		48	0	0.000	0.00	0.00
63	49	1900		49	0	0.000	0.00	0.00
64	50	1900		50	0	0.000	0.00	0.00
65	51	1900		51	0	0.000	0.00	0.00
66	52	800		52	0	0.000	0.00	0.00
67	53	800		53	0	0.000	0.00	0.00

Figure 111. Departure and arrival flow profile results, Example Calculation 26, offset = 30 sec

Figure 112 shows the arrival of the front of the platoon at the downstream intersection at t = 54 sec. Since the platoon arrives during green (the offset is set to 30 sec), the queue is zero during the time interval shown in the figure.

	A	B	C	D	E	F	G	H
12	Flow Profile Departing			Flow Profile Arriving				
13	Time i	DepFlowi		Time i	ArrFlowj	ArrFlowj	QueueStatus	Queue
67	53	800		53	0	0.000	0.00	0.00
68	54	800		54	375	0.104	0.00	0.00
69	55	800		55	676	0.188	0.00	0.00
70	56	800		56	917	0.255	0.00	0.00
71	57	800		57	1111	0.309	0.00	0.00
72	58	800		58	1267	0.352	0.00	0.00
73	59	800		59	1392	0.387	0.00	0.00
74	60	0		60	1492	0.414	0.00	0.00
75	61	0		61	1572	0.437	0.00	0.00
76	62	0		62	1637	0.455	0.00	0.00
77	63	0		63	1689	0.469	0.00	0.00

Figure 112. Departure and arrival flow profile results, Example Calculation 26, offset = 30 sec

Figure 113 shows the queue accumulation polygon for this case. The maximum queue of 2.3 vehicles occurs at t = 119 sec.

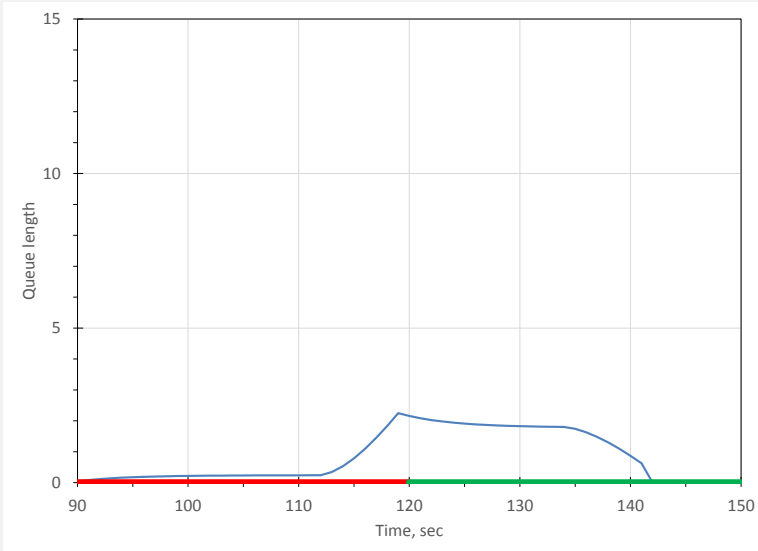


Figure 113. Queue accumulation polygon, Example Calculation 26, offset = 30 sec

But when the offset is set to zero, the platoon arrives during red. The queue accumulation polygon in Figure 114 shows the result. The maximum queue of 11.0 vehicles occurs at t = 89 sec.

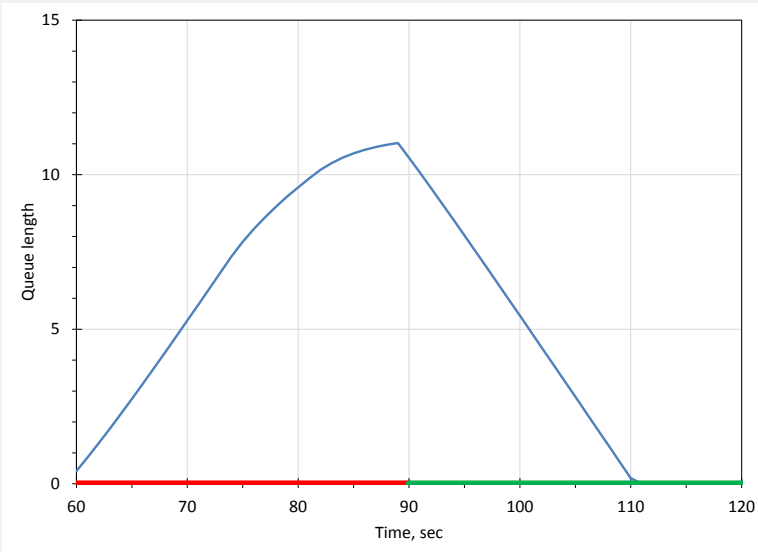


Figure 114. Queue accumulation polygon, Example Calculation 26, offset = 0 sec

Figure 115 shows the variation in the delay as a function of the offset, where the offset ranges from 0 to 60 sec. While the lowest delay value occurs at an offset of 25 sec, the queue accumulation polygon representing an offset of 30 sec shown in Figure 113 shows a delay near zero with most vehicles arriving during green. When the offset is set to zero, the delay is much larger as shown in Figure 114.

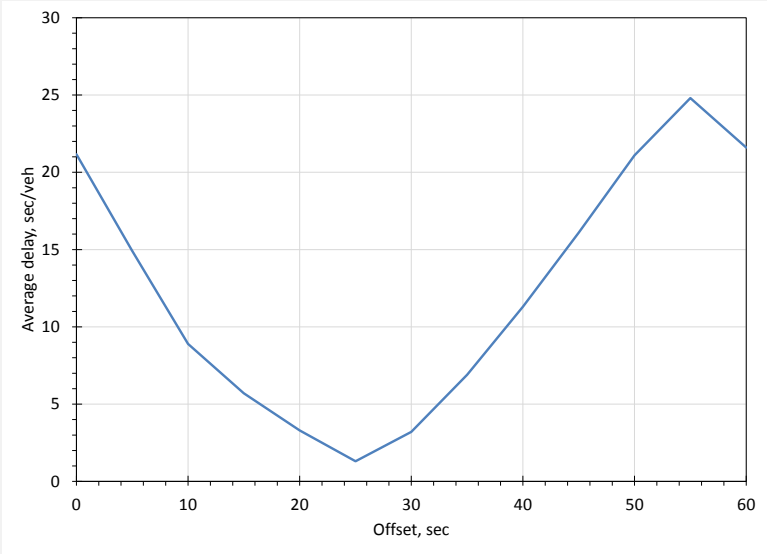


Figure 115. Average delay as a function of offset, Example Calculation 26

Scenario 7. Predicting Average Green Time Prediction under Actuated Control

The computational engine to predict green time under actuated control is based on the model presented in section 12 of this chapter. That section showed how the queue service time and the green extension time are predicted to produce the duration of green time. However, when considering two conflicting intersection approaches as in Scenario 7, the process becomes iterative. That is, the green time for one approach is predicted, and, based on this result, the green time for the other approach is predicted. After several iterations, the predicted value of green for each approach converges to a stable value, as we will soon see. Figure 116 shows the queue accumulation polygons for the movements controlled by phases 2 and 4. Figure 117 shows a graphical representation of the computational steps to predict the green times and cycle length. We assume that the demand on each approach is less than the approach capacity.

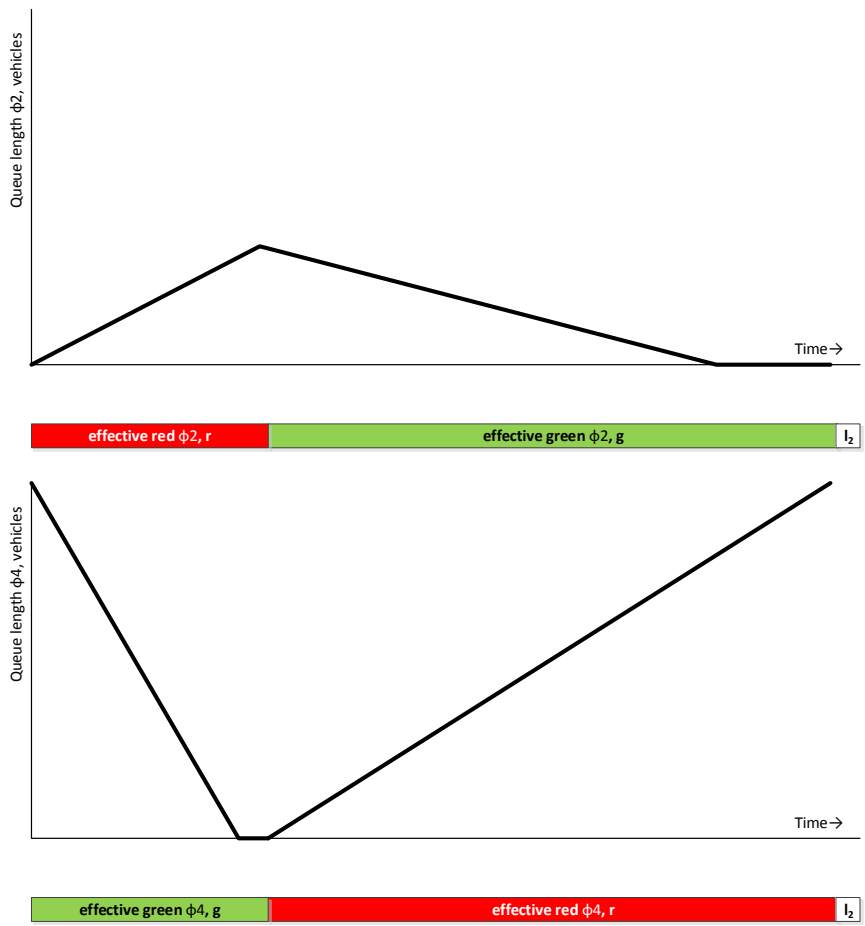


Figure 116. Example queue accumulation polygons for movements controlled by phases 2 and 4, Scenario 7

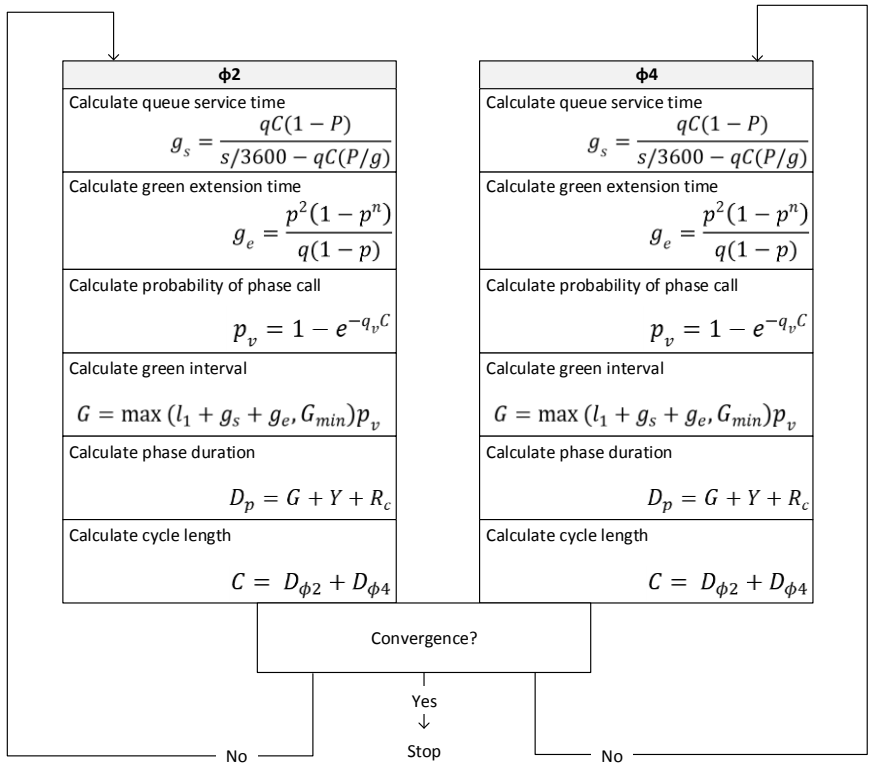


Figure 117. Flow chart to predict green times and cycle length for actuated control, Scenario 7

Requirements and Template

The computational engine for Scenario 7 will satisfy the following requirements:

1. Accepts the input values for the variables shown in Figure 118.
2. Calculates and writes the values as shown in Figure 119.
3. Uses VBA functions to calculate most values; these functions are documented in Table x.
4. Uses VBA code to automate the calculation process; this code is documented in Table x.

	A	B	C	D
1	**Input variables**	**Phase 2**	**Phase 4**	
2	Arrival rate, v			veh/hr
3	Proportion vehicles arriving on green, Pgreen			
4	Headway of bunched vehicles, Δ			veh/sec
5	Bunching factor, b			
6	Lost time, l_1			sec
7	Passage time, PT			sec
8	Detection zone length, L_{ds}			feet
9	Vehicle length, L_v			feet
10	Speed, S_a			mi/hr
11	Maximum green, G_{max}			sec
12	Minimum green, G_{min}			sec
13	Yellow time, Y			sec
14	Red clearance, R_c			sec
15	Saturation flow rate, s			veh/hr/green

Figure 118. Template for input variables to predict green times for actuated control, Scenario 7 computational engine

	A	B	C	D
17	**Calculated variables**	**Phase 2**	**Phase 4**	
18	Arrival rate, q			veh/sec
19	Arrival rate during red, q_r			veh/sec
20	Arrival rate during green, q_g			veh/sec
21	Proportion of free vehicles, φ			
22	Flow rate parameter, λ			veh/sec
23	Queue at end of red, QueueRed			veh
24	Queue service time, g_s			sec
25	Number of extension before max out, n			
26	ProbSubjectPhaseCalled, p_v			
27	Maximum allowable headway, MAH			sec
28	ProbGreenExtension, pge			
29	Green extension, g_e			sec
30	Effective change interval, Y + R_c			sec
31	Average green interval duration, Green			sec
32	Phase duration, D_p			sec
33				
34	**Final values**	**Phase 2**	**Phase 4**	
35	Cycle length, C			sec
36	Effective green, g			sec
37	Effective red, r			sec
38	Displayed green, G			sec
39	Displayed red, R			sec
40	Displayed yellow, Y			sec

Figure 119. Template for values to predict green times for actuated control, Scenario 7 computational engine

Example Formulas and Visual Basic Functions

Table 16 shows the Visual Basic code for the functions used in the computational engine for Scenario 7.

Table 16. Visual Basic functions, Scenario 7 computational engine

```
Public Function phi(b, delta, q)
phi = Exp(-b * delta * q)
End Function
```

```
Public Function lambda(phi, q, delta)
lambda = (phi * q) / (1 - delta * q)
End Function
```

```
Public Function QueueServiceTime(q, C, p, s, g, l1, Gmax, ge)
'q = arrival rate
'C = cycle length
'P = proportion of vehicles arriving on green
's = saturation flow rate
'g = effective green time
QST = (q * C * (1 - p)) / ((s / 3600) - q * C * (p / g))
If QST <= 0 Then
    QST = 0
ElseIf QST + ge + l1 > Gmax Then
    QST = Gmax - l1
End If

QueueServiceTime = QST

End Function
```

```
Public Function Qr(p, q, C, r)
Qr = (q * C * (1 - p)) / r
End Function
```

```
Public Function qg(p, q, C, g)
qg = (p * q * C) / g
End Function
```

```
Public Function NumberExtensionsBeforeMaxOut(qg, Gmax, gs, l1)
```

```
NumberExtensionsBeforeMaxOut = qg * (Gmax - (gs + l1))
If NumberExtensionsBeforeMaxOut < 0 Then NumberExtensionsBeforeMaxOut = 0

End Function
```

```
Public Function ProbSubjectPhaseCalled(q, C)
ProbSubjectPhaseCalled = 1 - Exp(-q * C)
End Function
```

```
Public Function ProbGreenExtension(phi, lambda, MAH, delta)
ProbGreenExtension = 1 - phi * Exp(-lambda * (MAH - delta))
End Function
```

```
Public Function MAH(PT, Lds, Lv, Sa)
MAH = PT + (Lds + Lv) / (1.47 * Sa)
End Function
```

```
Public Function GreenExtension(p, n, qg)
'Possible need to check if gs+l1>Gmax; if true, then ge is set to zero
GreenExtension = (p ^ 2 * (1 - p ^ n)) / (qg * (1 - p))
End Function
```

```
Public Function UniformDelay(v, r, gs, Qmax, Qendgreen, ge, C)
If Qendgreen <= 0 Then
    TotalDelay = 0.5 * (r + gs) * Qmax
ElseIf Qendgreen > 0 Then
    Area1 = r * Qmax * 0.5
    Area2 = (Qmax - Qendgreen) * (gs + ge) * 0.5
    Area3 = Qendgreen * (gs + ge)
    TotalDelay = Area1 + Area2 + Area3
End If
UniformDelay = TotalDelay / ((v / 3600) * C)

End Function
```

```
Public Function AvgGreenDuration(Gmin, Gmax, l1, gs, ge, pv)
If l1 + gs + ge < Gmin Then
    AvgGreenDuration = Gmin
ElseIf l1 + gs + ge < Gmax Then
    AvgGreenDuration = l1 + gs + ge
ElseIf l1 + gs + ge >= Gmax Then
    AvgGreenDuration = Gmax
End If

AvgGreenDuration = AvgGreenDuration * pv

End Function
```

```
Public Function QueueAtEndOfGreen(QueueRed, s, qg, Gmax, gs, ge, l1)
If QueueRed - (((s / 3600) - qg) * (gs)) <= 0 Then
    QueueAtEndOfGreen = 0
ElseIf QueueRed - (((s / 3600) - qg) * (gs)) > 0 Then
    QueueAtEndOfGreen = QueueRed - (((s / 3600) - qg) * (gs))
End If
End Function
```

The Visual Basic code that runs the procedure for Scenario 7 is given in Appendix 1 at the end of this chapter.

Example Calculation 27

This example shows the results for flow rates of 800 veh/hr for movement 2 and 100 veh/hr for movement 4 for Scenario 7. The input data are shown in Figure 120. The calculated values are shown in Figure 121. The final cycle length is calculated to be 25.2 sec.

	A	B	C	D
1	**Input variables**	**Phase 2**	**Phase 4**	
2	Arrival rate, v	800	100	veh/hr
3	Proportion vehicles arriving on green, P	0.5	0.5	
4	Headway of bunched vehicles, Δ	1.5	1.5	veh/sec
5	Bunching factor, b	0.6	0.6	
6	Lost time, l_1	2	2	sec
7	Passage time, PT	2.5	2.5	sec
8	Detection zone length, L_{ds}	22	22	feet
9	Vehicle length, L_v	20	20	feet
10	Speed, S_a	30	30	mi/hr
11	Maximum green, G_{max}	50	50	sec
12	Minimum green, G_{min}	5	5	sec
13	Yellow time, Y	3	3	sec
14	Red clearance, R_c	2	2	sec
15	Saturation flow rate, s	1900	1900	veh/hr/green

Figure 120. Input data, Example Calculation 27

	A	B	C	D
17	**Calculated variables**	**Phase 2**	**Phase 4**	
18	Arrival rate, q	0.222	0.028	veh/sec
19	Arrival rate during red, q_r	0.213	0.019	veh/sec
20	Arrival rate during green, q_g	0.232	0.051	veh/sec
21	Proportion of free vehicles, φ	0.819	0.975	
22	Flow rate parameter, λ	0.273	0.028	veh/sec
23	Queue at end of red, QueueRed	4.455	0.557	veh
24	Queue service time, g_s	15.061	1.168	sec
25	Number of extension before max out, n	7.641	2.395	
26	ProbSubjectPhaseCalled, p_v	1.000	0.672	
27	Maximum allowable headway, MAH	3.452	3.452	sec
28	ProbGreenExtension, pge	0.519	0.077	
29	Green extension, g_e	2.145	7.919	sec
30	Effective change interval, Y + R_c	5.000	5.000	sec
31	Average green interval duration, Green	19.203	7.447	sec
32	Phase duration, D_p	24.206	16.087	sec
33				
34	**Final values**	**Phase 2**	**Phase 4**	
35	Cycle length, C	40.3	40.3	sec
36	Effective green, g	19.2	11.1	sec
37	Effective red, r	21.1	29.2	sec
38	Displayed green, G	19.2	7.4	sec
39	Displayed red, R	18.1	29.8	sec
40	Displayed yellow, Y	3	3	sec

Figure 121. Calculated values for Example Calculation 27

In this example, five iterations were required to meet the convergence test. Table 17 shows these results. The final cycle length was 40.29 sec, decreasing from the 65.48 sec in iteration 1. The green time for phase 2 decreased from 40.24 sec in iteration 1 to 19.2 sec in iteration 5. The final green time for phase 4 is 7.45 sec.

Table 17. Intermediate values for green time and cycle length, Example Calculation 27

Iteration number	Green time for phase 2, sec	Green time for phase 4, sec	Cycle Length, sec
1	40.239	14.294	65.481
2	25.641	9.279	46.717
3	20.396	7.728	41.029
4	19.204	7.407	40.097
5	19.203	7.447	40.293

Example Calculation 28

This example shows the results for flow rates of 1050 veh/hr for movement 2 and 400 veh/hr for movement 4 for Scenario 7. The input data are shown in Figure 122. The calculated values are shown in Figure 123. The final cycle length is calculated to be 61.1 sec.

In this example, eleven iterations are required to meet the convergence test. During the calculation process, the cycle length decreases from 79.5 sec in iteration 1 to 61.4 sec in iteration 11.

	A	B	C	D
1	**Input variables**	**Phase 2**	**Phase 4**	
2	Arrival rate, v	1050	400	veh/hr
3	Proportion vehicles arriving on green, P	0.5	0.5	
4	Headway of bunched vehicles, Δ	1.5	1.5	veh/sec
5	Bunching factor, b	0.6	0.6	
6	Lost time, l_1	2	2	sec
7	Passage time, PT	2.5	2.5	sec
8	Detection zone length, L_{ds}	22	22	feet
9	Vehicle length, L_v	20	20	feet
10	Speed, S_a	30	30	mi/hr
11	Maximum green, G_{max}	50	50	sec
12	Minimum green, G_{min}	5	5	sec
13	Yellow time, Y	3	3	sec
14	Red clearance, R_c	2	2	sec
15	Saturation flow rate, s	1900	1900	veh/hr/green

Figure 122. Input values, Example Calculation 28

	A	B	C	D
17	**Calculated variables**	**Phase 2**	**Phase 4**	
18	Arrival rate, q	0.292	0.111	veh/sec
19	Arrival rate during red, q_r	0.353	0.074	veh/sec
20	Arrival rate during green, q_g	0.249	0.221	veh/sec
21	Proportion of free vehicles, φ	0.769	0.905	
22	Flow rate parameter, λ	0.399	0.121	veh/sec
23	Queue at end of red, QueueRed	8.960	3.413	veh
24	Queue service time, g_s	32.104	11.143	sec
25	Number of extension before max out, n	3.953	8.163	
26	ProbSubjectPhaseCalled, p_v	1.000	0.999	
27	Maximum allowable headway, MAH	3.452	3.452	sec
28	ProbGreenExtension, pge	0.647	0.285	
29	Green extension, g_e	1.881	2.250	sec
30	Effective change interval, Y + R_c	5.000	5.000	sec
31	Average green interval duration, Green	35.985	15.376	sec
32	Phase duration, D_p	40.985	20.393	sec
33				
34	**Final values**	**Phase 2**	**Phase 4**	
35	Cycle length, C	61.4	61.4	sec
36	Effective green, g	36.0	15.4	sec
37	Effective red, r	25.4	46.0	sec
38	Displayed green, G	36.0	15.4	sec
39	Displayed red, R	22.4	43.0	sec
40	Displayed yellow, Y	3	3	sec

Figure 123. Calculated values, Example Calculation 28

Table 18. Intermediate values for green times and cycle length, Example Calculation 28

Iteration number	Green2, sec	Green4, sec	Cycle, sec
1	50.000	19.521	79.522
2	42.832	18.849	71.684
3	40.585	16.929	67.520
4	38.378	16.486	64.873
5	37.490	15.920	63.422
6	36.780	15.730	62.523
7	36.454	15.552	62.021
8	36.220	15.479	61.715
9	36.104	15.422	61.542
10	36.026	15.395	61.438
11	35.985	15.376	61.378

15. Summary

In this chapter we explored the models on which the HCM capacity analysis method for signalized intersections is based. We reviewed how a signalized intersection operates in the field, and the factors on which models to predict the capacity of an intersection could be used. We reviewed the concepts and terminology used to define movements and phases, including the ring barrier diagram. We learned about the traffic control process diagram and how it Is used to represent the timing processes for actuated signal controllers. We formulated the basic D/D/1 queuing model and how it is represented using the flow profile diagram, the cumulative vehicle diagram, and the queue accumulation polygon. We saw how seven simplified scenarios could represent various aspects of the signalized intersection analysis method to make it easier to understand. Finally, we constructed four computational engines to test various aspects of the models.

16. Glossary

Table 19. Terms used in this chapter

Term	Definition
Capacity flow rate	The maximum flow rate that would be observed departing from a signalized intersection if green were displayed for an entire hour. It is also the product of the saturation flow rate and the effective green ratio.
Concurrency group	
Critical movement analysis	Tool used to assess the sufficiency of the capacity of a signalized intersection.
Cumulative vehicle diagram	The running total of the number of vehicles that have arrived at and departed from a signalized intersection over time.
D/D/1 queuing model	Deterministic arrival pattern, deterministic service pattern, and one service channel
Effective green ratio	The proportion of the hour that is effectively available for a given movement.
Effective green time	The green time available to serve traffic.
Flow profile diagram	Represents the flow rate over time for both the arrivals and departures at a signalized intersection.
Flow ratio	Ratio of the volume for a given movement to the saturation flow rate for that movement.
Gap out	Occurs when both the minimum green timer and the passage timer expire.
Green extension time	The time that the green is extended ..
Length of queue	The difference between the number of vehicles that have arrived at and departed from a signalized intersection at any point in time.
Lost time	The time during the cycle that can't be utilized by vehicles.
Maximum green time	Maximum duration that the signal display will remain green after a call has been received on a conflicting phase.
Max out	Occurs when the maximum green timer expires.
Minimum green time	Minimum time that the signal will remain green no matter what else occurs.
Movement	The direction of travel from its origin and the turning maneuver that a vehicle completes to get to its destination. A movement is also categorized by any restriction that may be placed on it; for example, protected movement.
Passage time	Maximum time that a detector can remain unoccupied before the passage timer expires.
Phase	A timing unit that controls one or more compatible movements at a signalized intersection.
Queue accumulation polygon	The length of queue at any point in time at a signalized intersection.
Ring	A sequence of movements that must be served one after the other.
Ring barrier diagram	Tool used to define those movements that are compatible and can be served concurrently.
Saturation flow rate	The maximum number of vehicles that can pass by the stop line on a lane or approach if the green indication is displayed continuously for an hour.
Saturation headway	The headway between vehicles in the departing queue at a signalized intersection.
Server	In queuing theory, the first vehicle position at the stop bar.
Timing stage	Interval of time during which no signal displays change.
Traffic control process diagram	A functional representation of an actuated traffic control system.
Uniform flow	When the arrival headway between vehicles is constant.
Uniform arrivals	When the headways are constant between vehicle arrivals.
Uniform delay	The delay experienced by vehicles at a signalized intersection when the arrival pattern is uniform.
Volume capacity ratio (or degree of saturation)	Ratio of the volume to the capacity for a given movement.

Table 20. Variables used in this chapter

Variable	Description	Units
B = 0.6	Bunching factor	
c	Capacity	veh/hr
C	Cycle length	sec
d_{avg}	Average uniform delay	sec/veh
d_{int}	Average delay for the intersection	sec/veh
D_p	Duration of phase	sec
D_t	Total uniform delay	veh-sec
d_t	Time step duration	s/steps
e	Extension of effective green	sec
F	Smoothing factor	
g	Effective green time	sec

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Variable	Description	Units
G	Displayed green time	sec
g_e	Green extension time	sec
G_{max}	Maximum green time	sec
g_{so}	Time for opposing queue to clear	sec
g_s	Queue service time	sec
h_s	Saturation headway	sec
L	Lost time per cycle	sec
L_v	Detection length of vehicle	ft
L_{ds}	Length of the stop-line detection zone	ft
l_1	Start-up lost time	sec
MAH	Maximum allowable headway	sec
N	Average number of green extensions before phase terminates	
n	Number of extensions between the serving of the queue and the when the maximum green time is reached.	
n_g	Arrival count during green	
P	Proportion of vehicles arriving during the green indication	
PT	Passage time	sec
p_v	Probability of phase being called	
q=3600/v	Arrival flow rate	veh/sec
q_g	Arrival flow rate during effective green	veh/sec
q_d	Arrival flow rate for downstream lane group	veh/s
$q'_{a\|u,j}$	Arrival flow rate in time step j at downstream intersection from upstream source u	veh/step
Q_r	Queue size at end of effective red	
q_r	Arrival flow rate during effective red	veh/sec
$q'_{u,i}$	Departure flow rate in time step I at upstream source u	veh/step
RC or R_c	Red clearance time	sec
R_p	Platoon ratio	
s	Saturation flow rate	veh/hr
s_p	Saturation flow rate of a permitted left turn movement	veh/hr
t' (j)	Time step associated with platoon arrival time	
t'	Platoon arrival time	steps
t_o	Time that detector is occupied	sec
$t'_R = t_R/d_t$	Segment running time	steps
t_R	Segment running time	?
t_f	Follow-up headway	sec
t_c	Critical headway	sec
t_L	Total lost time for a phase	sec
t_u	Time that detector is not occupied (unoccupancy time)	sec
v	Arrival rate or demand flow rate	veh/sec
v_g	Flow rate during green	veh/hr
v_o	Opposing demand flow rate	veh/hr
v_r	Flow rate during red	veh/hr
X	Volume/capacity ratio or degree of saturation	veh/hr
X_c	Critical volume-capacity ratio for intersection	-
Y	Flow ratio	-
Y	Yellow time (change interval)	sec
$\Delta = 1.5s$	Headway of bunched vehicle	sec
λ	Flow rate parameter	
p	Probability that a headway between two vehicles after the queue has been served is less than the MAH	
φ	Proportion of free (unbunched) vehicles	

17. References

The primary model to calculate the capacity of a signalized intersection is based on a D/D/1 queuing model: deterministic arrivals, deterministic departures, and a single service channel. Webster and Cobb describe to method to estimate delay at a fixed time signalized intersection that has come to be known as Webster's method. Van Hurdle provides an excellent description of the model in the following paper. Transportation Research Circular 212 describes the basic model as well as the critical movement analysis for assessing the sufficiency of signalized intersection capacity. Tom Urbanik and his co-authors describe the operation of an actuated traffic control system, useful background for those interested in understanding the process in greater detail. Rahmi Akcelik describes the method used to estimate the green time for an actuated traffic signal system. Bonneson, Pratt, and Vandehey describe various models included in the HCM method to calculate green time at an actuated signalized intersection and the method to determine the arrival pattern of traffic at a signalized intersection as filtered by an upstream signalized intersection.

F.V. Webster and B.M. Cobbe, "Traffic Signals". Her Majesty's Stationary Office, London, England, 1966.

V.F. Hurdle. "Signalized Intersection Delay Models: A Primer for the Uninitiated". Transportation Research Record 971. Transportation Research Board, 19xx.

Transportation Research Board. Interim Materials on Highway Capacity. Circular 212. Washington, D. C., TRB, 1980.

Tom Urbanik, et.al. "Signal Timing Manual". NCHRP Report 812. National Cooperative Highway Research Program. Transportation Research Board, Washington, D.C., 2015.

R. Akcelik. "Estimate of Green Times and Cycle time for Vehicle Actuated Signals. Presented at the 73rd Annual Meeting of the Transportation Research Board, Washington, D.C., January 1994.

James A Bonneson, Michael P Pratt, Mark Vandehey. "Predicting the performance pf Automobile Traffic on Urban Streets", Project 3-79, Final Report, TTI/TAMU, KAI, Purdue, January 2008. Appendix C. Procedure for estimating arrival flow profile and Appendix D. Procedure for estimating coordinated-actuated phase duration.

Appendix 1. VBA Code for Scenario 7 Computational Engine

```
Sub Analyze()
'This sub calculates the green durations for phases 2 and 4 for actuated signal control
'Written by Michael Kyte, 2016.03.18
'Modified 2016.03.19

'Define variables
'Arrival Rate, v
'Proportion vehicles arriving on green, Pgreen
'Headway of bunched vehicles, delta
'Bunching factor, b
'Lost Time, l1
'Passage Time, PT
'Detection zone length, Lds
'Vehicle Length, Lv
'Speed , Sa
'Maximum green, Gmax
'Minimum green, Gmin
'Yellow Time, Y
'Red clearance, Rc
'Saturation flow rate, s

'Declare variables
Dim v2, Pgreen2, delta2, b2, l12, PT2, Lds2, Lv2, Sa2, Gmax2, Gmin2, Y2, Rc2, s2 As Double
Dim v4, Pgreen4, delta4, b4, l14, PT4, Lds4, Lv4, Sa4, Gmax4, Gmin4, Y4, Rc4, s4 As Double

'Intermediate Values
'Cycle Length, C
'Effective green, g
'Effective red, r

'Declare variables
Dim C As Double
Dim g2, r2, g4, r4 As Double

'Calculated variables
'Arrival Rate, q
'Arrival rate during red, qr
'Arrival rate during green, qg
'Proportion of free vehicles, phi
'Flow rate parameter, lambda
'Queue at end of red, QueueRed
'Queue service time, gs
'Number of extension before max out, n
'ProbSubjectPhaseCalled , pv
'Maximum allowable headway, MAH
'ProbGreenExtension , pge
'green extension, ge
'Effective change interval, YRc
'Average green interval duration, Green, AvgGreenDuration
'Phase Duration, Dp

Dim q2, qr2, qg2, phi2, lambda2, QueueRed2, gs2, n2, pv2, MAH2, pge2, ge2, YRc2,
AvgGreenDuration2, Dp2 As Double
Dim q4, qr4, qg4, phi4, lambda4, QueueRed4, gs4, n4, pv4, MAH4, pge4, ge4, YRc4,
AvgGreenDuration4, Dp4 As Double

'Final Values
'Cycle Length, C
'Effective green, g
'Effective red, r
'Displayed green, g
'Displayed red, Red
'Displayed yellow, Y

'Functions called from Module 1
'Public Function phi(b, delta, q) - proportion of free vehicles
'Public Function lambda(phi, q, delta) - flow rate parameter
'Public Function QueueServiceTime(q, C, p, s, g, l1, Gmax, ge)
```

```
'Public Function QueueRed(p, q, C, r) - queue at end of red
'Public Function qg(p, q, C, g) - flow rate during green
'Public Function NumberExtensionsBeforeMaxOut(qg, Gmax, gs, l1)
'Public Function ProbSubjectPhaseCalled(q, C)
'Public Function ProbGreenExtension(phi, lambda, MAH, delta)
'Public Function MAH(PT, Lds, Lv, Sa) - maximum allowable headway
'Public Function GreenExtension(p, n, qg)
'Public Function UniformDelay(v, r, gs, QueueRed, Qendgreen, ge, C)
'Public Function AvgGreenDuration(Gmin, Gmax, l1, gs, ge, pv)
'Public Function QueueAtEndOfGreen(QueueRed, s, qg, Gmax, gs, ge, l1)

'Set calculated values to blank in rows 18 through 60
Range("b18").Select
    For i = 0 To 15
        For j = 0 To 1
            ActiveCell.Offset(i, j).Value = ""
        Next j
    Next i

Range("a44").Select
    For i = 0 To 15
        For j = 0 To 3
            ActiveCell.Offset(i, j).Value = ""
        Next j
    Next i

'Set initial values for cycle length, green, and red
    C = 100
    g2 = 50
    g4 = 50
    r2 = 50
    r4 = 50
    e = 2

'Read input values for phase 2
v2 = Range("b2").Value
Pgreen2 = Range("b3").Value
delta2 = Range("b4").Value
b2 = Range("b5").Value
l12 = Range("b6").Value
PT2 = Range("b7").Value
Lds2 = Range("b8").Value
Lv2 = Range("b9").Value
Sa2 = Range("b10").Value
Gmax2 = Range("b11").Value
Gmin2 = Range("b12").Value
Y2 = Range("b13").Value
Rc2 = Range("b14").Value
s2 = Range("b15").Value

'Read input values for phase 4
v4 = Range("c2").Value
Pgreen4 = Range("c3").Value
delta4 = Range("c4").Value
b4 = Range("c5").Value
l14 = Range("c6").Value
PT4 = Range("c7").Value
Lds4 = Range("c8").Value
Lv4 = Range("c9").Value
Sa4 = Range("c10").Value
Gmax4 = Range("c11").Value
Gmin4 = Range("c12").Value
Y4 = Range("c13").Value
Rc4 = Range("c14").Value
s4 = Range("c15").Value

'Calculate static variable values for phase 2
q2 = v2 / 3600
phi2 = phi(b2, delta2, q2)
lambda2 = lambda(phi2, q2, delta2)
MAH2 = MAH(PT2, Lds2, Lv2, Sa2)
```

```
pge2 = ProbGreenExtension(phi2, lambda2, MAH2, delta2)
YRc2 = Y2 + Rc2

'Calculate static variable values for phase 4
q4 = v4 / 3600
phi4 = phi(b4, delta4, q4)
lambda4 = lambda(phi4, q4, delta4)
MAH4 = MAH(PT4, Lds4, Lv4, Sa4)
pge4 = ProbGreenExtension(phi4, lambda4, MAH4, delta4)
YRc4 = Y4 + Rc4

'Set initial value for printing intermediate values in row 44 and following
k = 0

'Loop to check if values have converged.
Do

    'Cycle length from previous iteration
    Clast = C

    'Calculate iterative (dynamic) variable values for phase 2
    qr2 = qr(Pgreen2, q2, C, r2)
    qg2 = qg(Pgreen2, q2, C, g2)
    QueueRed2 = qr2 * r2
    gs2 = QueueServiceTime(q2, C, Pgreen2, s2, g2, l12, Gmax2, ge2)
    n2 = NumberExtensionsBeforeMaxOut(q2, Gmax2, gs2, l12)
    pv2 = ProbSubjectPhaseCalled(q2, C)
    ge2 = GreenExtension(Pgreen2, n2, qg2)
    g2 = gs2 + ge2 + e
    Green2 = AvgGreenDuration(Gmin2, Gmax2, l12, gs2, ge2, pv2)
    Dp2 = l12 + gs2 + ge2 + Y2 + Rc2

    'Calculate iterative (dynamic) variable values for phase 4
    qr4 = qr(Pgreen4, q4, C, r4)
    qg4 = qg(Pgreen4, q4, C, g4)
    QueueRed4 = qr4 * r4
    gs4 = QueueServiceTime(q4, C, Pgreen4, s4, g4, l14, Gmax4, ge4)
    n4 = NumberExtensionsBeforeMaxOut(q4, Gmax4, gs4, l14)
    pv4 = ProbSubjectPhaseCalled(q4, C)
    ge4 = GreenExtension(Pgreen4, n4, qg4)
    g4 = gs4 + ge4 + e
    Green4 = AvgGreenDuration(Gmin4, Gmax4, l14, gs4, ge4, pv4)
    Dp4 = l14 + gs4 + ge4 + Y4 + Rc4

    'Calculate current iteration C and r value
    C = Dp2 + Dp4
    r2 = C - g2
    r4 = C - g4

    'Write current values of green phase lengths for phases 2 and 4 and cycle length
    'Initial value of k set to 0
    k = k + 1
    Range("a43").Select
    ActiveCell.Offset(k, 0).Select
    ActiveCell.Value = k
    ActiveCell.Offset(0, 1) = Green2
    ActiveCell.Offset(0, 2) = Green4
    ActiveCell.Offset(0, 3) = C

Loop Until Abs(Clast) - C < 0.1

'Write final values to spreadsheet cells for phase 2
Range("b18").Value = q2
Range("b19").Value = qr2
Range("b20").Value = qg2
Range("b21").Value = phi2
Range("b22").Value = lambda2
Range("b23").Value = QueueRed2
Range("b24").Value = gs2
Range("b25").Value = n2
Range("b26").Value = pv2
```

```
Range("b27").Value = MAH2
Range("b28").Value = pge2
Range("b29").Value = ge2
Range("b30").Value = YRc2
Range("b31").Value = Green2
Range("b32").Value = Dp2

Range("b35").Value = C
Range("b36").Value = g2
Range("b37").Value = r2
Range("b38").Value = Green2
Range("b39").Value = C - Green2 - Y2
Range("b40").Value = Y2

'Write final values to spreadsheet cells for phase 4
Range("c18").Value = q4
Range("c19").Value = qr4
Range("c20").Value = qg4
Range("c21").Value = phi4
Range("c22").Value = lambda4
Range("c23").Value = QueueRed4
Range("c24").Value = gs4
Range("c25").Value = n4
Range("c26").Value = pv4
Range("c27").Value = MAH4
Range("c28").Value = pge4
Range("c29").Value = ge4
Range("c30").Value = YRc4
Range("c31").Value = Green4
Range("c32").Value = Dp4

Range("c35").Value = C
Range("c36").Value = g4
Range("c37").Value = r4
Range("c38").Value = Green4
Range("c39").Value = C - Green4 - Y4
Range("c40").Value = Y4

End Sub
```