CHAPTER 2. CAPACITY OF TWO-WAY STOP-CONTROLLED INTERSECTIONS

1. Overview

In this chapter we will explore the models on which the HCM capacity analysis method for two-way stop-controlled (TWSC) intersections are based. Table 1 shows a roadmap to the material presented in this chapter. We'll start in section 2 with a discussion of how TWSC intersections operate in the field, particularly the interaction between drivers on the major street and those on the stop-controlled approaches. In section 3, we'll identify the important factors in this interaction that will help us to formulate the models to predict the capacity of a traffic stream. The models that we will formulate in sections 4 and 5 are based on "simplified scenarios", scenarios in which only the most important traffic and geometric factors are considered. By focusing only on these factors, you will develop a basic understanding of the operation of a TWSC intersection, one that can later be built upon as the more complex conditions found in the real world are considered.

Table 1. Roadmap to chapter 2

Overview	Simplified Scenarios	Learning in Depth	Closing
1. Overview 2. What do we see in the field 3. Formulating the model	4. Intersection of two one-way streets 5. T-intersection	6. Calculating the critical headway and follow-up headway 7. Building computational engine and exploring the model	8. Summary 9. Glossary 10. References

You will study two such scenarios. The first scenario is based on the intersection of two one-lane one-way streets. The second scenario is based on a T-intersection with two-way traffic on the major street but only left turning traffic on the stop-controlled approach. In both scenarios, we assume a traffic stream consisting only of passenger cars and no pedestrians to impede the flow of automobile traffic.

You will see that the operation of a TWSC intersection is based on the decisions made by minor stream drivers (those on the stop-controlled approaches) about when it is safe to enter the intersection. Two model parameters represent these decisions in the models, the critical headway and the follow-up headway. You will learn how to calculate these parameters in Section 6.

You will build a computational engine in section 7, based on the two scenarios described above. You can use the computational engine (in the form of a spreadsheet) to study the predictions of capacity that the models make under a range of traffic flow conditions. These predictions will help you to understand when two-way stop control might be an effective control strategy and under what conditions other types of control should be considered.

Section 8 presents a summary of the chapter. Section 9 presents a list of all terms and variables used in the chapter and provides a definition or description for each. A list of references is presented in section 10.

2. What Do We Observe in the Field?

A TWSC intersection provides priority to vehicles on the major street while requiring vehicles on the minor street to stop before entering the intersection. Minor street drivers look for gaps in the major street traffic stream that are large enough for them to safely enter the intersection and complete their maneuver.

Since we are interested in predicting the capacity of a traffic stream, let's consider what drivers see and do as they approach a TWSC intersection. Drivers on the major street traveling through or turning right proceed through the intersection without stopping. Drivers turning left from the major street may be delayed if there are oncoming through or right-turning vehicles; these left turning vehicles may need to wait to find a suitable gap in this oncoming traffic stream before completing their maneuver through the intersection.

Drivers on the stop-controlled approaches must stop when they arrive at the stop line and wait for suitable gaps in the major street traffic stream. They must also negotiate "whose turn it is" with drivers on the other stop-controlled approach, jointly deciding who can use the next available gap. The capacity of each minor stream is thus dependent on (1) the traffic flow rates of the higher priority streams as well as (2) on the behavior of the minor stream drivers and the gaps that they require to complete a specific maneuver.

Other factors also limit the capacity of a traffic stream, such as pedestrians crossing the street or heavy vehicles that are slower to accelerate than passenger cars. But while these factors are often present in the field, we will only consider simplified scenarios in which there are no pedestrians and a traffic stream consisting only of passenger cars.

So, why do we study the capacity of a TWSC intersection? Sometimes, we want to determine what type of control is optimal for an intersection given demand and geometric conditions. We might also want to know when it is justified to change from stop sign control to signal control. Or, perhaps there is a driveway from a shopping center or residential development that enters an arterial and we want to know whether stop sign control will be adequate. The models that we will develop in this chapter can help to answer these questions.

3. Formulating the Model

We will use a queuing model to represent the traffic flow conditions on a stop-controlled approach of a TWSC intersection based on the conditions that we've observed in the field. This queuing model consists of the following parts:

- Vehicles arrive at the intersection, on both the major and minor streets, in a random manner, with headways between vehicles following a negative exponential distribution.
- Vehicles on the stop-controlled approaches require a minimum headway in the major traffic stream to safely enter the intersection and complete their maneuver. This minimum headway is called the critical headway.
- If there is a continuous queue on a stop-controlled approach, and a large enough gap between vehicles on the major street, the headway between vehicles entering the intersection from this stop-controlled approach using this gap is called the follow-up headway.
- There is a hierarchy among the traffic streams. For example, through and right-turning vehicles on the major street always have priority over vehicles in the other traffic streams. By contrast, left-turning minor stream vehicles must give way to vehicles in all other traffic streams.

The process described above is known as a gap acceptance process. The gap acceptance process can be observed in other traffic flow situations. Examples include vehicles merging from an on-ramp to a freeway mainline, vehicles passing on a two-lane highway, and vehicles making permitted left turns at a signalized intersection.

Since we use both terms, gap and headway, to describe what seems like the same thing, it is important to define clearly what each term means. A gap is what drivers see and make judgements about: it is the distance or time between the back bumper of one car and front bumper of following car in the major street traffic stream. The headway that we measure corresponding to this gap is the time between the front bumpers of the first and second cars.

To illustrate the gap acceptance process consider the sequence of events shown in Figure 1. The sequence begins with a queue of three vehicles on the NB (stop-controlled) approach and a vehicle on the major street (EB approach, vehicle A) entering the intersection. The next vehicle to arrive on the major street approach, Vehicle B, enters the intersection (event 2) x.x sec later. The headway between vehicles A and B (2.0 sec) is too small for vehicle 1 on the stop-controlled approach to enter the intersection. The driver of vehicle 1 is said to reject the gap between vehicles A and B. In event 3, we note that vehicle 1 has just cleared the intersection when vehicle C enters the intersection. Vehicle 1 is said to have accepted the gap between vehicle B and vehicle C. While we don't have enough data in this simple illustration to make a more precise estimate, we do know that the critical headway for vehicle 1 is greater than 2.0 sec but less than x.x sec. The process for estimating the critical headway is described in Section 6 of this chapter.

Now consider the sequence of events shown in Figure 2. The sequence also begins with a queue of three vehicles on the NB approach, and a vehicle (vehicle A) on the major street entering the intersection. In event 2, vehicle 1 enters the intersection, followed by vehicle 2 (event 3) and vehicle 3 (event 4). Finally, in event 5, the next major street vehicle (Vehicle B) enters the intersection.

The gap accepted by vehicles 1, 2, and 3 is defined by the headway between vehicles A and B, which we calculate below to be 9.0 sec..

$$h = clock\ time\ (event\ 5) - clock\ time(event\ 1)$$

$$h = 9.0 - 0.0 = 9.0\ sec$$

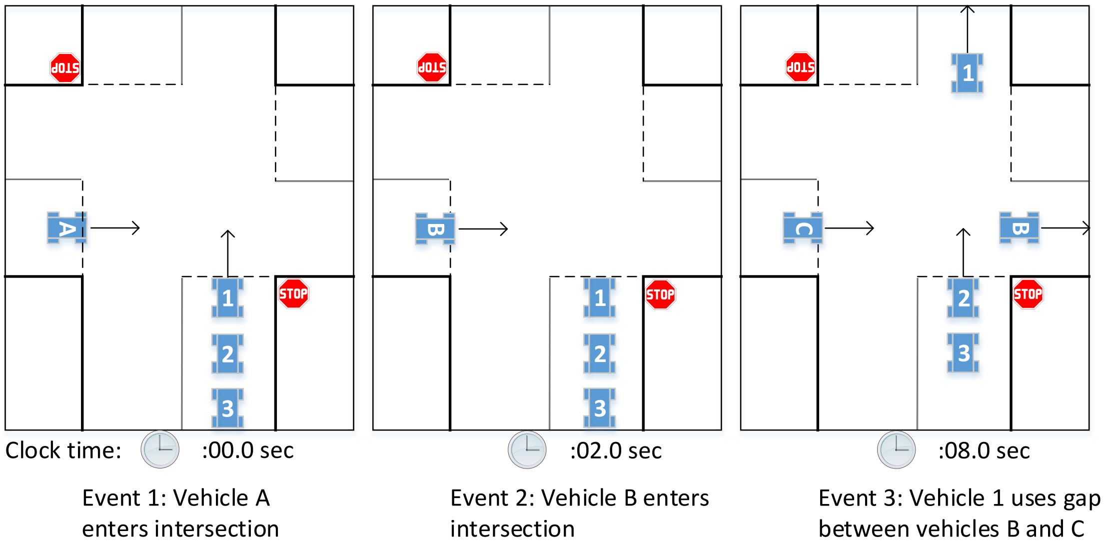

Figure 1. Minor street vehicle rejecting and accepting gaps

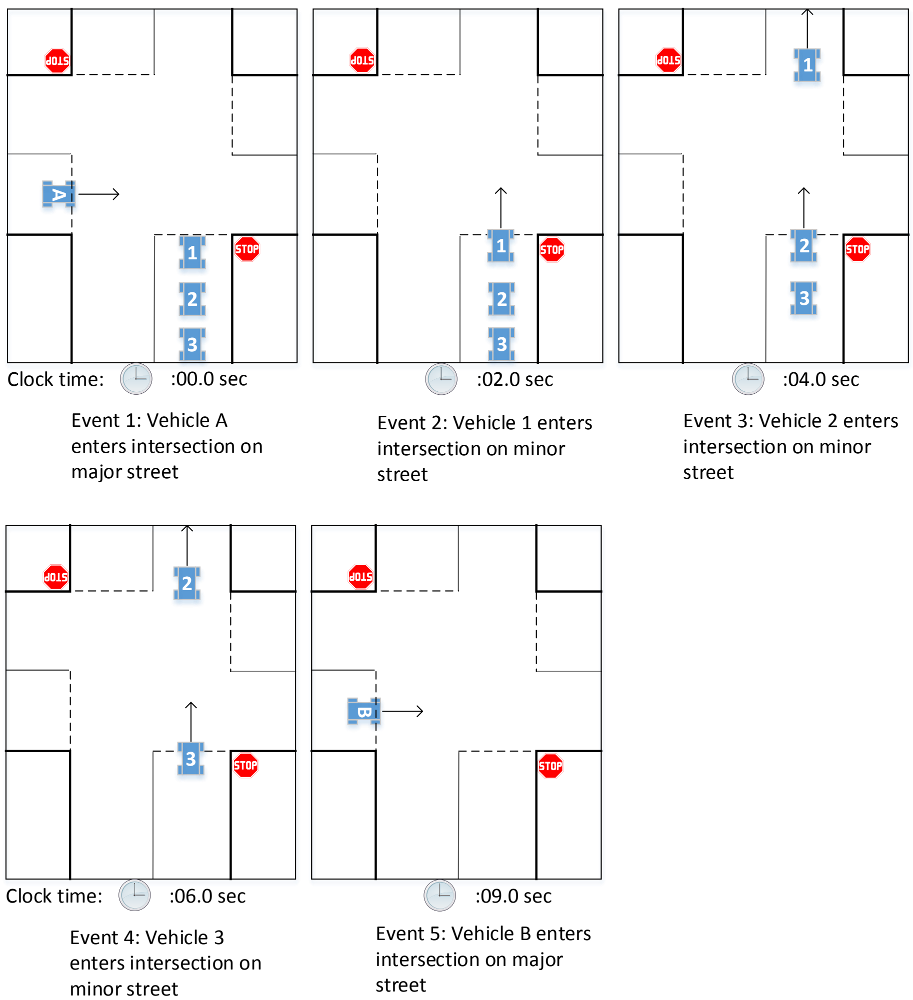

Figure 2. Minor street vehicles using same major street gap

We know that this headway is larger than the critical headway since it was used by more than one minor street vehicle. But we can calculate the follow up headway since more than one vehicle from the stop-controlled approach that was part of a continuous queue used the same major street gap. We calculate the follow up headway to be 2.0 seconds, based on these two measurements.

$$t_f = clock\ time\ (event\ 3) - clock\ time\ (event\ 2)$$

$$t_f = 4.0 - 2.0 = 2.0\ sec$$

$$t_f = clock\ time\ (event\ 4) - clock\ time\ (event\ 3)$$

$$t_f = 6.0 - 4.0 = 2.0\ sec$$

In summary, the gap acceptance model for determining the capacity of a minor traffic stream at a TWSC intersection is based on the following concepts:

- Drivers in a minor traffic stream require a minimum headway between vehicles in the major traffic stream to safely enter the intersection. This headway is called the critical headway. As we will see later in the chapter, the critical headway is estimated statistically based on the number and size of gaps that are accepted and rejected by minor stream drivers.
- When gaps are large enough, more than one minor stream vehicle may use the same gap. The headway between consecutive minor stream vehicles using the same major stream gap during conditions of continuous queuing in the minor stream is called the follow-up headway.

In the next section, we'll develop a scenario that implements this model formulation for the intersection of two one-lane one-way streets.

4. Scenario 1: Intersection of Two One-Way Streets

Let's consider an intersection of two one-lane, one-way streets, with through movements only. Vehicles on one of the streets, the major street, always have the right-of-way. Vehicles on the minor street, controlled by a stop sign, must wait for a suitable gap in the major street traffic stream before entering the intersection. We'll call this Scenario 1. In this section, we'll study the conditions on the minor street approach and develop a model to predict the capacity of the approach. This capacity depends directly on the major street traffic volume as well as the behavioral characteristics of drivers on the minor street. Said another way, we will consider both the availability of gaps on the major traffic stream and the usefulness of these gaps to minor stream drivers. See Figure 3.

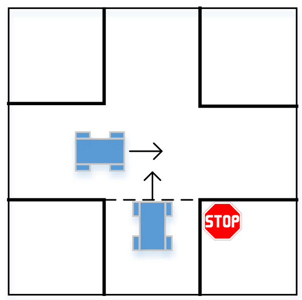

Figure 3. Scenario 1

Availability of Gaps

We start by representing the availability of gaps in the major traffic stream. And, remember, we are assuming that the arrival of major street vehicles can be represented by a random process. Two common statistical distributions are used to represent random processes:

- The Poisson distribution is a discrete distribution, often used to describe how many random events will occur during a specific time interval.
- The negative exponential distribution is a continuous distribution that is used to represent the time between the occurrences of these random events.

Suppose we have a single lane on the major street on which traffic is flowing in one direction. Further assume that the operation of each vehicle is independent of any of the other vehicle in the lane. If the mean flow rate is λ (veh/sec), the probability of observing x vehicles during a specified time period t is given by the Poisson distribution:

Equation 1

$$P[x] = \frac{(\lambda t)^x}{x!} e^{-\lambda t}$$

For these conditions, what is the probability that we observe no vehicles during the interval t? The probability that x is zero (no vehicles observed during the interval) is given by Equation 2.

Equation 2

$$P[x = 0] = \frac{(\lambda t)^0}{0!} e^{-\lambda t} = e^{-\lambda t}$$

Another way of looking at the condition described in Equation 2 is: what is the probability that we will see a headway in the traffic stream of at least t seconds? This can be represented by the density function for the negative exponential distribution.

Equation 3

$$P[h \geq t] = e^{-\lambda t}$$

Example Calculation 1

Suppose the mean arrival rate of vehicles on the major street is 400 veh/hr or .111 veh/sec. What is the probability that we will observe a headway greater than or equal to 4 sec? What is the probability that we will see a gap less than 4 sec?

From Equation 3:

$$P[h \geq t] = e^{-\lambda t}$$

The probability that we will observe a headway greater than or equal to 4 sec is:

$$P[h \geq 4] = e^{-(.111)(4)} = .642$$

And the probability that we will observe a headway less than 4 sec is given by:

$$P[h < t] = 1 - P[h \geq t] = 1 - .642 = .359$$

Usefulness of Gaps

Next, we'll look at the usefulness of the available major street gaps to drivers on the minor or stop-controlled approach. The behavior of the minor street drivers is represented by two parameters, the critical headway and the follow-up headway. Headways that are greater than or equal to the critical headway are large enough for the minor street driver to safely enter the intersection and are thus considered useful. Headways that are less than the critical headway are rejected by a minor street driver and are not considered useful.

We will further assume that drivers are both homogeneous (they behave the same way at any location) and consistent (they behave the same way every time). This means that we can use one value for t_c and t_f for all drivers in a given traffic stream.

Usefulness of Headway Ranges

The capacity of the minor street approach is calculated by combining these concepts. Consider ranges of major street headways and the number of minor street vehicles that can use each range. We know that if a major street headway is less than the critical headway, no minor street vehicle can use the gap that creates the headway. We also know that if the headway is greater than or equal to t_c, then at least one minor street vehicle can use the gap. But we also know if the gap is large enough, more than one minor street vehicle can use the gap. Table 2 shows the number of vehicles can use each headway ranges, covering the cases of no vehicles, one vehicle, and two vehicles, and for the general case of n vehicles.

Table 2. Number of vehicles that can use each headway range

Headway range, t_h	Number of vehicles able to use range
$t_h < t_c$	0
$t_c \leq t_h < t_c + t_f$	1
$t_c + t_f \leq t_h < t_c + 2t_f$	2
…	
$t_c + (n-1)t_f \leq t_h < t_c + nt_f$	n

Likelihood of Headway Range Occurring

The next step is to calculate the likelihood that each headway range will occur. We know that the probability that a headway in the major traffic stream is less than t_c, and thus too small to be usable by any vehicle, is given by:

Equation 4

$$P[h < t_c] = 1 - e^{-\lambda t_c}$$

where λ is the mean arrival rate of the major traffic stream.

The probability that a gap is large enough for only one vehicle is given by the difference between the probability of a headway less than or equal to t_c and the probability of a headway less than or equal to t_c plus t_f:

Equation 5

$$P\left[t_c \leq h < t_c + t_f\right] = e^{-\lambda t_c} - e^{-\lambda(t_c + t_f)}$$

In general, the probability that a gap is large enough to be used by n vehicles is given by:

Equation 6

$$P\left[t_c + (n-1)t_f \leq h < t_c + nt_f\right] = e^{-\lambda(t_c + (n-1)t_f)} - e^{-\lambda(t_c + nt_f)}$$

The probability of a given headway range occurring is given column 3 in Table 3.

Table 3. Probability of occurrence of given headway range

Headway range, t_g	Number of vehicles able to use range	Probability of range occurrence
$t_h < t_c$	0	$1 - e^{-\lambda t_c}$
$t_c \leq t_h \leq t_c + t_f$	1	$e^{-\lambda t_c} - e^{-\lambda(t_c + t_f)}$
$t_c + t_f \leq t_h \leq t_c + 2t_f$	2	$e^{-\lambda(t_c + t_f)} - e^{-\lambda(t_c + 2t_f)}$
…		
$t_c + (n-1)t_f \leq t_h \leq t_c + nt_f$	n	$e^{-\lambda(t_c + (n-1)t_f)} - e^{-\lambda(t_c + nt_f)}$

Capacity of Minor Stream

We can combine the concepts of gap availability, gap (and headway range) usefulness, and the likelihood of headway range occurrence to estimate the capacity of a stream or movement. We note that capacity is based on the distribution of gaps in the major stream, minor stream driver judgement in selecting gaps, and follow-up headway required by drivers in a queue in the minor stream.

The number of minor stream vehicles that can enter the intersection during one hour is the capacity of the stop-controlled approach, given the flow rate on the major street. This capacity flow rate can be written as the sum of the products of the probability that a given headway range will occur and the number of vehicles that can use this range. The fourth column in Table 4 shows the probable number of vehicles that can use each headway range. Each row in this column is the product of columns 2 and 3.

Table 4. Probable number of vehicles using headway range

Headway range, t_g	Number of vehicles able to use range	Probability of range occurrence	Probable number of vehicles using range
$t_h < t_c$	0	$1 - e^{-\lambda t_c}$	0
$t_c \le t_h \le t_c + t_f$	1	$e^{-\lambda t_c} - e^{-\lambda(t_c+t_f)}$	$1\left(e^{-\lambda t_c} - e^{-\lambda(t_c+t_f)}\right)v_c$
$t_c + t_f \le t_h \le t_c + 2t_f$	2	$e^{-\lambda(t_c+t_f)} - e^{-\lambda(t_c+2t_f)}$	$2\left(e^{-\lambda(t_c+t_f)} - e^{-\lambda(t_c+2t_f)}\right)v_c$
…			
$t_c + (n-1)t_f \le t_h \le t_c + nt_f$	n	$e^{-\lambda(t_c+(n-1)t_f)} - e^{-\lambda(t_c+nt_f)}$	$n\left(e^{-\lambda(t_c+(n-1)t_f)} - e^{-\lambda(t_c+nt_f)}\right)v_c$

Equation 7 shows the sum of column 4, an infinite series, to be the probable number of vehicles that can enter the intersection in an hour. This result is the capacity of the approach.

Equation 7

$$c = 0 + 1\left(e^{-\lambda t_c} - e^{-\lambda(t_c+t_f)}\right)v_c + 2\left(e^{-\lambda(t_c+t_f)} - e^{-\lambda(t_c+2t_f)}\right)v_c + \cdots + n\left(e^{-\lambda(t_c+(n-1)t_f)} - e^{-\lambda(t_c+nt_f)}\right)v_c$$

$$c = v_c\left(e^{-\lambda t_c} + e^{-\lambda(t_c+t_f)} + e^{-\lambda(t_c+2t_f)} + \cdots + e^{-\lambda(t_c+(n-1)t_f)} + e^{-\lambda(t_c+nt_f)}\right)$$

where v_c is the major street flow rate in veh/hr.

This can also be written in a shorter form as shown in Equation 8.

Equation 8

$$c = v_c \sum_{n=0}^{\infty} e^{-\lambda(t_c+nt_f)}$$

The sum of the series is given by Equation 9:

Equation 9

$$c = \frac{v_c e^{-v_c t_c/3600}}{1 - e^{-v_c t_f/3600}}$$

where v_c is the flow rate on the major street and is equal to 3600λ.

Example Calculation 2

Research has shown that for the minor street through movement t_c = 6.5 sec and t_f = 4.0 sec. What is the capacity of the minor street approach if the major street flow rate is 400 veh/hr or .111 veh/sec?

We can use Equation 9 to determine the capacity flow rate.

$$c = \frac{v_c e^{-v_c t_c/3600}}{1 - e^{-v_c t_f/3600}} = \frac{400e^{-400(6.5)/3600}}{1 - e^{-400(4.0)/3600}} = 541\ veh/hr$$

We can also illustrate the capacity calculation using the process shown in Table 4. Table 5 shows the probable number of vehicles using ranges up to 62.5 sec. The sum of the probable number of vehicles using the fourteen headway ranges is 540.3 veh/hr, virtually equal to the 541 veh/hr calculated above. As we consider additional headway ranges, the cumulative sum of the probable number of vehicles approaches the calculated value of 541 veh/hr. Figure 4 shows that the greater the number of ranges considered, the closer the cumulative number of vehicles approaches the capacity prediction of 541 veh/hr.

Table 5. Cumulative number of vehicles using headway range

Headway range (sec)	Number of vehicles able to use range	Probability of range occurrence	Probable number of vehicles using range	Cumulative number of probable vehicles
< 6.5	0	$1 - e^{-\lambda t_c} = .514$	0	0.0
6.5 - 10.5	1	$e^{-\lambda t_c} - e^{-\lambda(t_c+t_f)} = .174$	$1\left(e^{-\lambda t_c} - e^{-\lambda(t_c+t_f)}\right)v_c$ = 1(.174)(400) = 69.6	69.6
10.5 - 14.5	2	$e^{-\lambda(t_c+t_f)} - e^{-\lambda(t_c+2t_f)} = .112$	$2\left(e^{-\lambda(t_c+t_f)} - e^{-\lambda(t_c+2t_f)}\right)v_c$ = 2(.112)(400) = 89.3	159.0
14.5 – 18.5	3	$e^{-\lambda(t_c+2t_f)} - e^{-\lambda(t_c+3t_f)} = .072$	$3\left(e^{-\lambda(t_c+2t_f)} - e^{-\lambda(t_c+3)}\right)v_c$ = 3(.072)(400) = 86.0	244.9
18.5 – 22.5	4	$e^{-\lambda(t_c+3t_f)} - e^{-\lambda(t_c+4t_f)} = .046$	$4\left(e^{-\lambda(t_c+3t_f)} - e^{-\lambda(t_c+4t_f)}\right)v_c$ =(4)(.046)(400) = 73.5	318.4
22.5 – 26.5	5	$e^{-\lambda(t_c+4t_f)} - e^{-\lambda(t_c+5t_f)} = .030$	$5\left(e^{-\lambda(t_c+4t_f)} - e^{-\lambda(t_c+5t_f)}\right)v_c$ = (5)(.030)(400) = 58.9	377.4
…				
26.5 – 30.5	6	0.019	45.4	422.8
30.5 – 34.5	7	0.012	34.0	456.7
34.5 – 38.5	8	0.008	24.9	481.6
38.5 – 42.5	9	0.005	18.0	499.6
42.5 – 46.5	10	0.003	12.8	512.4
46.5 – 50.5	11	0.002	9.0	521.4
50.5 – 54.5	12	0.001	6.3	527.7
54.5 – 58.5	13	0.001	4.4	532.1
58.5 – 62.5	14	0.001	3.0	535.2
…				
	Sum	1.000	541	

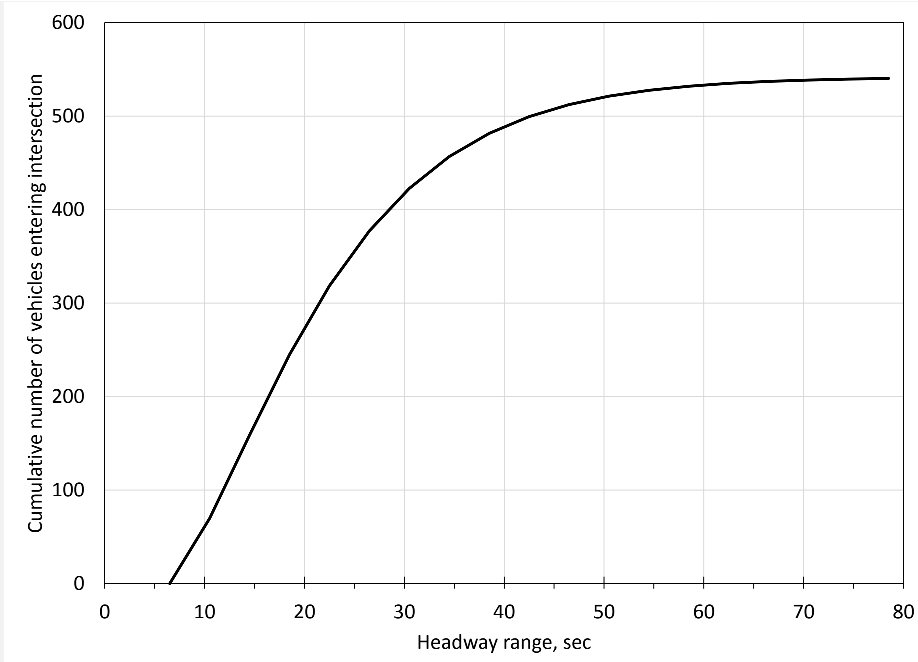

Figure 4. Cumulative number of vehicles entering intersection as function of headway range

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Example Calculation 3

How does the capacity of the minor street approach vary with changes in the major street flow rate? As before, we assume t_c = 6.5 sec and t_f = 4.0 sec.

When v_c = 0:

$$c \rightarrow \frac{3600}{t_f} = \frac{3600}{4.0} = 900\ veh/hr$$

When v_c = 200 veh/hr (or a mean headway in the major street traffic stream of 18 sec):

$$c = \frac{v_c e^{-v_c t_c/3600}}{1 - e^{-v_c t_f/3600}} = \frac{200 e^{-200(6.5)/3600}}{1 - e^{-200(4.0)/3600}} = \frac{139.3803}{.1993} = 699\ veh/hr$$

When v_c = 900 veh/hr (or a mean headway in the major street traffic stream of 4 sec):

$$c = \frac{v_c e^{-v_c t_c/3600}}{1 - e^{-v_c t_f/3600}} = \frac{900 e^{-900(6.5)/3600}}{1 - e^{-900(4.0)/3600}} = \frac{177.2295}{.6321} = 280\ veh/hr$$

Figure 5 shows the variation of the minor street capacity with changes in the conflicting flow rate on the major street. When the conflicting flow rate approaches zero, the capacity flow rates approaches 3600/t_f, or 900 veh/hr. As the conflicting flow rate increases, the capacity decreases. While the model never predicts a zero capacity, from a practical perspective, the probability of finding a useful gap (one for which the headway exceeds the critical headway) diminishes greatly when the flow rate approaches and then exceeds 1000 veh/hr. At a major street flow of 1000 veh/hr, the probability that a headway exceeds t_c is 0.15; at 1500 veh/hr, the probability is .07. This diminishing probability is shown in Figure 6.

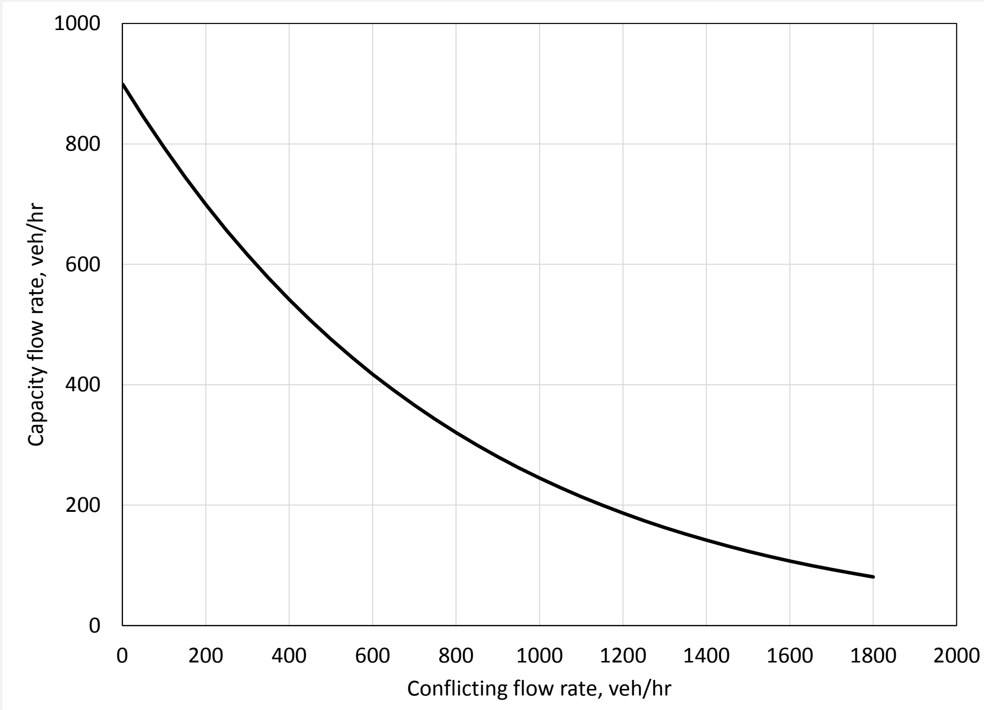

Figure 5. Capacity flow rate as function of conflicting flow rate

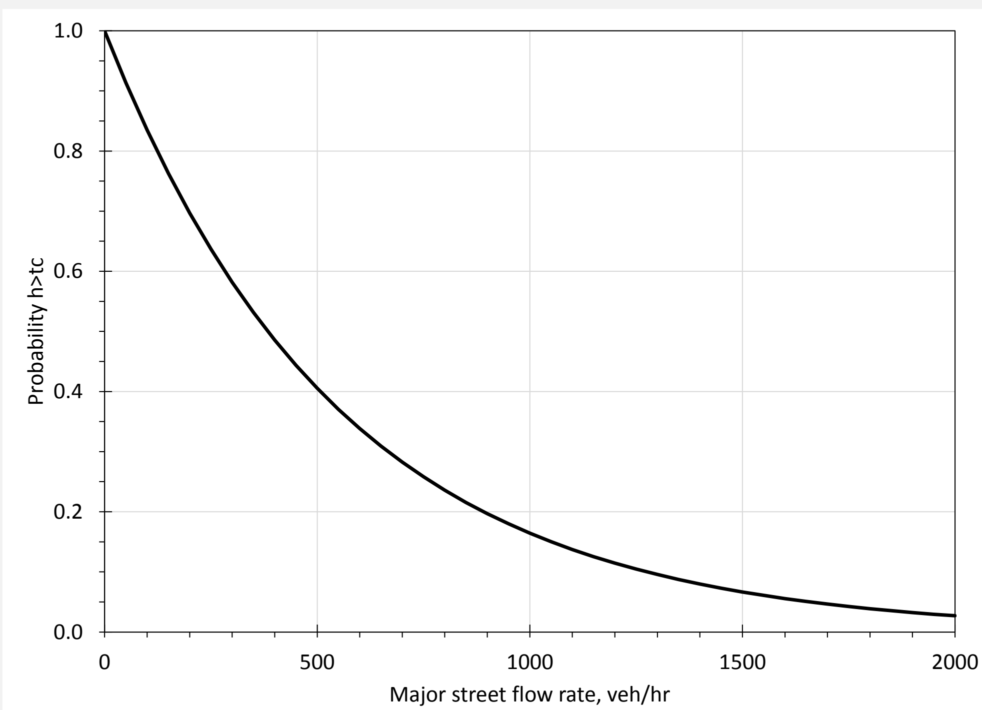

Figure 6. Probability of headway exceeding critical headway as function of major street flow rate

5. Scenario 2: T-Intersection

Let's now consider a second simplified scenario, a T-intersection with one lane on each of its three approaches. We will only consider three movements, the EBTH and WBLT movements on the major street and the NBLT movement on the minor street. The movements are numbered according to the scheme used in the HCM, as shown in Figure 7.

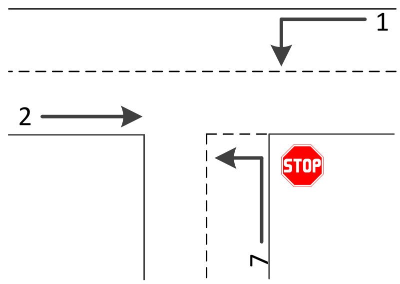

Figure 7. Scenario 2

Potential Capacity

We need to distinguish between two terms, the potential capacity and the movement capacity. The movement capacity will be considered in the next section. The potential capacity for movements 4 and 7 is calculated based on Equation 8 and shown below as Equation 10 and Equation 11, respectively.

Equation 10

$$c_{p,4} = \frac{v_{c,4} e^{-v_{c,4} t_{c4}/3600}}{1 - e^{-v_{c,4} t_{f4}/3600}}$$

Equation 11

$$c_{p,7} = \frac{v_{c,7} e^{-v_{c,7} t_{c7}/3600}}{1 - e^{-v_{c,7} t_{f7}/3600}}$$

Let's now consider how to calculate the conflicting flow rate $v_{c,i}$ for each movement. The conflicting flow for movement 4 is equal to the volume for movement 2, the only movement that conflicts with (or has a higher priority than) movement 4.

Equation 12

$$v_{c,4} = v_2$$

However, research has shown that the conflicting flow for movement 7 is equal to the movement 2 volume plus twice the movement 4 volume. Movement 7 drivers apparently react to the movement 4 volume to a degree greater than that represented simply by the volume. Equation 13 gives the conflicting flow for movement 7.

Equation 13

$$v_{c,7} = v_2 + 2v_4$$

Movement Capacity and Impedance

There is an obvious hierarchy among these three traffic movements. One traffic movement, movement 2, doesn't stop for any other movement. It always has the right-of-way and is called rank 1. At the other extreme, movement 7 must always stop for the other two movements. It is called rank 3. In the middle, movement 4 must stop for or yield to movement 2, but it has priority over movement 7. It is called rank 2.

This hierarchy results in an important concept that we will call impedance. Gaps that could be usable to movement 7 vehicles (rank 3) may not be available because a rank 2 vehicle may use the same gap. For example, let's say there is a headway of 8 sec between movement 2 vehicles, usually large enough for a waiting movement 7 vehicle to safely enter the intersection. But if a movement 4 vehicle is also waiting for a gap in movement 2 vehicle, the movement 7 vehicle would not be able to use this gap.

To formalize the concept of impedance, note that the probability of a movement 4 vehicle waiting to make a left turn is the traffic intensity (or ratio of the volume to the capacity) of that movement:

Equation 14

$$\rho_4 = \frac{\lambda_4}{\mu_4} = \frac{v_4}{c_4}$$

And, the probability that a movement 4 vehicle is not present, what we call a queue-free state for this movement, is given by:

Equation 15

$$p_{0,4} = 1 - \frac{v_4}{c_4}$$

The impedance of movement 4 on movement 7 drivers reduces the potential capacity for movement 7. This reduced capacity, called the movement capacity, is the product of its potential capacity and the probability that movement 4 is operating in a queue-free state, as given by Equation 16.

Equation 16

$$c_{m,7} = c_{p,7}\left(1 - \frac{v_4}{c_4}\right)$$

Or,

Equation 17

$$c_{m,7} = \left(\frac{v_{c,7}e^{-v_{c,7}t_c/3600}}{1 - e^{-v_{c,7}t_f/3600}}\right)\left(1 - \frac{v_4}{c_4}\right)$$

Example Calculation 4

Let's consider an example calculation to show the effects of movement hierarchy and the concept of impedance. The following conditions are given:

Table 6. Given conditions for Example Calculation 4

Movement	Volume	Rank	t_c	t_f
2	600	1	-	-
4	100	2	4.1	2.2
7	50	3	7.1	3.5

The conflicting flow for movement 4 is:

$$v_{c,4} = v_2 = 600\ veh/hr$$

The potential capacity for movement 4 is:

$$c_{p,4} = \frac{v_{c,4}e^{-v_{c,4}t_c/3600}}{1-e^{-v_{c,4}t_f/3600}} = \frac{600e^{-600(4.1)/3600}}{1-e^{-600(2.2)/3600}} = \frac{302.9586}{.3070} = 987\ veh/hr$$

This is also the movement capacity for movement 4:

$$c_{m,4} = c_{p,4} = 987\ veh/hr$$

The conflicting flow for movement 7 is:

$$v_{c,7} = v_2 + 2v_4 = 600 + 2(100) = 800\ veh/hr$$

The potential capacity for movement 7 is:

$$c_{p,7} = \frac{v_{c,7}e^{-v_{c,7}t_c/3600}}{1-e^{-v_{c,7}t_f/3600}} = \frac{(600+2*100)e^{-800(7.1)/3600}}{1-e^{-800(3.5)/3600}} = \frac{165.11467?}{.5406} = 306\ veh/hr$$

We now have to consider the impedance of movement 4 on movement 7. That is, gaps that would have been available to movement 7 vehicles that are actually used by movement 4 vehicles. Impedance is the proportion of time that a minor movement is blocked or impeded by a higher priority minor movement. The movement capacity for movement 7 is the product of the potential capacity for movement 7 and the proportion of time that movement 4 operates in a queue-free state.

The proportion of time that a queue is present for movement 4 is given by:

$$\frac{v_4}{c_{m,4}} = \frac{100}{987} = 0.10$$

The probability of a queue-free state is given by:

$$p_{0,4} = 1 - \frac{v_4}{c_{m,4}} = 1 - 0.1013 = .8987$$

The movement capacity for movement 7 is:

$$c_{m,7} = p_{0,4}c_{p,7} = (0.8987)(306) = 275$$

Example Calculation 5

Assume the same conditions as given in Example Calculation 4. Determine the effect of changing the WBLT volume on the NBLT capacity by varying the WBLT volumes from 0 to 1200 veh/hr.

Table 7 shows the volume for movement 4 for a range from zero to 600 veh/hr. The resulting conflicting flow for movement 7, the potential capacity for movement 7, the probability of a queue-free state for movement 4, and the movement capacity for movement 7 are also given in the table. As the volume for movement 4 (the WBLT movement) increases, the probability of a queue free-state for that movement decreases. Thus, the movement capacity for movement 7 decreases. Figure 8 illustrates these results for a range of movement 4 volumes from zero to 1200 veh/hr. Figure 9 shows how the volume-to-capacity ratio for movement 7 increases as the WBLT volume (movement 4) increases.

Table 7. Results for Example Calculation 5

v_4	$v_{c,7}$	$c_{p,7}$	$p_{0,4}$	$c_{m,7}$
0	600	416	1.000	416
50	700	357	0.949	338
100	800	306	0.899	275
150	900	262	0.848	222
200	1000	224	0.797	178
250	1100	191	0.747	143
300	1200	163	0.696	114
350	1300	140	0.645	90
400	1400	119	0.595	71
450	1500	101	0.544	55
500	1600	86	0.493	43
550	1700	74	0.443	33
600	1800	63	0.392	25

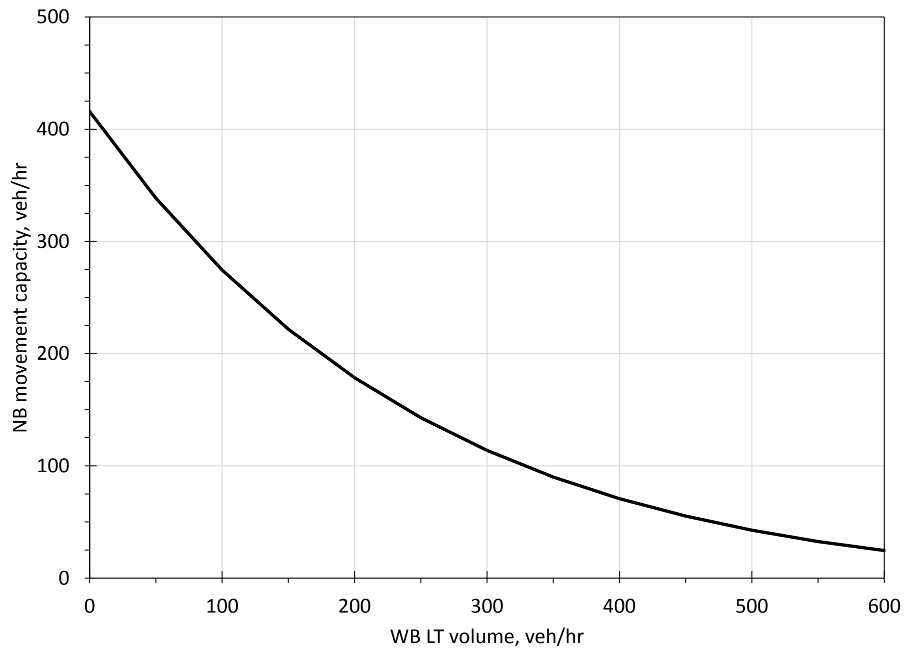

Figure 8. Movement capacity for movement 4, Example Calculation 5

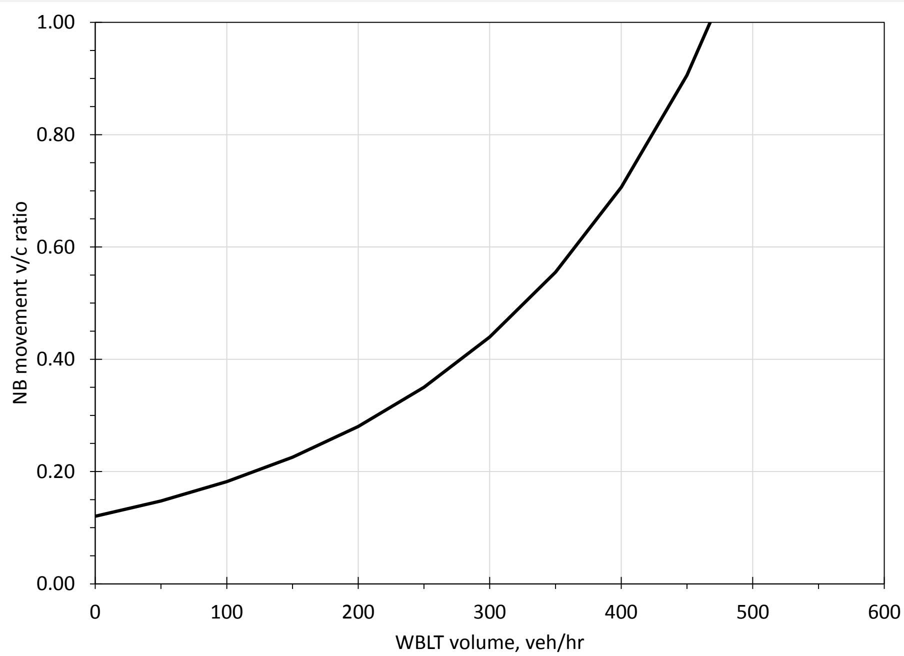

Figure 9. Volume-capacity ratio for movement 7, Example Calculation 5

6. Calculating the Critical Headway and Follow-Up Headway

The critical headway and the follow-up headway define the behavior of drivers at a TWSC intersection when we use a gap acceptance model. The critical headway is the minimum time interval in the major stream traffic stream that allows intersection entry for one minor street vehicle. The follow up headway is the time between the departure of one vehicle from the minor street and the departure of the next vehicle using the same major street headway, under conditions of continuous queuing on the minor street. In this section, we will learn how to use field data to calculate both parameters.

As we noted earlier, a driver actually observes the gap between vehicles when making his or her decision to enter the intersection. This gap is the time between the passage of the rear of one vehicle by a specific point and the arrival of the front of the next vehicle at that same point. However, gaps are difficult to measure consistently. It is standard practice to measure the headway, which is the time between the fronts of two consecutive vehicles in the traffic stream passing the same point. So while we often talk about the gap that a driver sees we refer to the headway as the relevant parameter that we use.

Accepted and Rejected Gaps

Let's look at an example of a T-intersection as shown in Figure 10 to illustrate how we estimate the critical headway. There are four movements, or traffic streams, at this intersection, including movements 2, 3, and 5 on the major street and movement 7 on the minor street, as shown in Figure 10.

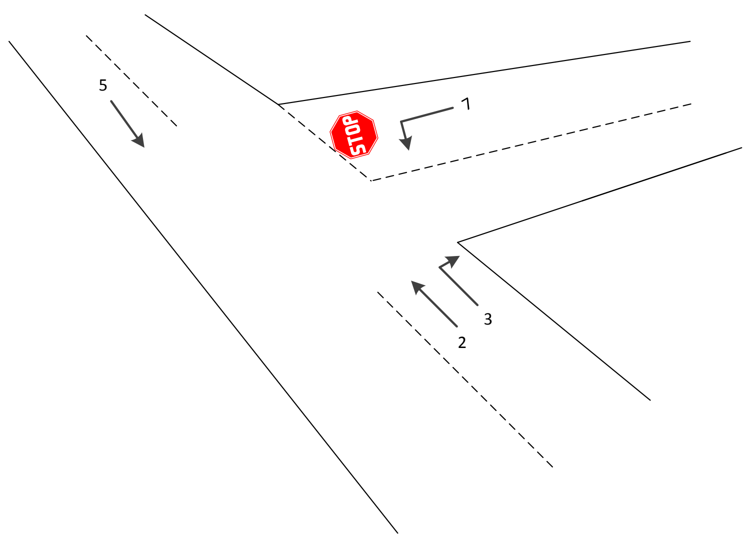

Figure 10. Movement numbers for example intersection

We will look at the time that three types of events occur: (1) vehicles on the major street entering and passing through the intersection, and vehicles on the minor street (2) arriving at the end of a queue and then (3) arriving at the stop line. In this example, we will consider event data collected at an intersection over a period of three minutes to illustrate some of the issues in determining the critical headway and the follow-up headway. From the standpoint of drivers at the minor street stop line, some gaps will be too small to be usable, while other gaps will be large enough to be usable by one or more minor stream drivers.

Table 8 shows the time that each event occurred during this three-minute period. The first six columns show the event number and the time that vehicles on the major street (movements 5, 2, and 3) pass through the central point of the intersection, what we might call the conflict point. The last four columns show the event number and the times that the minor street vehicles (movement 7) arrive at the stop line (the server) and enter the intersection (depart from the server). The time format shown in the table is minutes, seconds, and tenths of seconds.

Table 8. Events for example intersection

Vehicles arrive at conflict point on major street						Minor street vehicles, Movement 7			
Movement 5		Movement 2		Movement 3		Arrive server		Depart server	
Event	Time	Event	Time	Event	Time	Event	Time	Event	Time
1	00:15.4	1	00:24.5	1	00:26.7	1	00:05.4	2	00:09.3
2	00:20.2	2	00:37.3	2	00:44.3	3	00:11.9	4	00:28.6
3	00.22.6	3	00:46.6	3	00:55.3	5	00:32.9	6	01:04.0
4	00:25.4	4	00:50.4	4	01:12.0	7	01:09.1	8	01:11.1
5	00:26.2	5	01.02.8	5	01:15.0	9	01:17.6	10	01:39.9
6	00:28.3	6	01:26.8	6	01:19.4	11	01:42.4	12	01.47.3
7	00:45.0	7	01:45.6	7	01:24.6	13	01:50.8	14	01:54.2
8	00:48.7	8	01:52.6	8	01:30.9	15	01:57.4	16	02:04.3
9	00:58.6	9	01:57.8	9	02:13.5	17	02:08.4	18	02:10.9
10	01:32.6	10	02:02:2			19	02:18.8	20	02:19.8
11	01:34.8								
12	01:39.1								
13	01:44.9								
14	02:40.9								

Let's first illustrate a rejected gap. Figure 11 shows two events, Event 2 and Event 3 for movement 5. These events define the beginning and ending of a headway between two vehicles in the major street traffic stream. We note that the same two vehicles are waiting at the stop line during both events. This implies that the first vehicle in line on the stop-controlled approach rejects this gap. The duration of the headway represented by this gap is 22.6 sec – 20.2 sec, or 2.4 sec.

Figure 11. Two events for example intersection

Figure 12 shows a time line for these events. For the first 30 sec of the observation period, the figure shows the times that the major street vehicles pass the conflict point (the center of the intersection), and when each minor street vehicle arrives at the stop line and then enters the intersection. The minor street vehicle that arrives at the stop line at t = 11.9 sec waits at the stop line until t = 28.6 sec before it enters the intersection. It rejects seven gaps until finally accepting one. As represented in the figure, one of the gaps is rejected by the vehicles, shown as the headway between events 2 and 3.

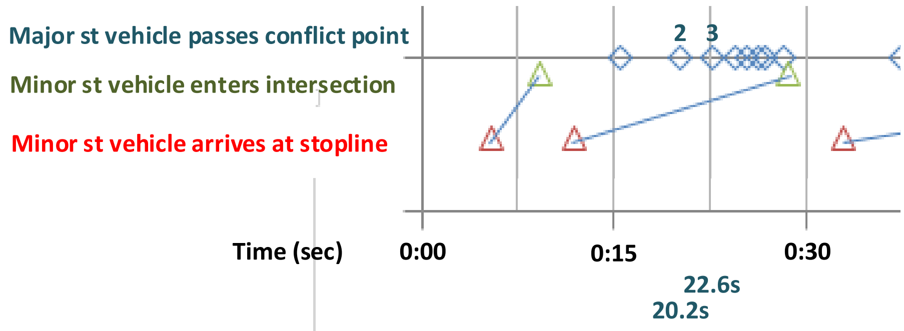

Figure 12. Time line for vehicle events

Let's next consider a series of events that illustrate an accepted gap. Figure 13 shows the four events of interest: (a) the arrival of a minor street vehicle, (b) the passage of a major street vehicle past the conflict point, (c) the entry of the minor street vehicle into the intersection, and (d) the passage of the next major street vehicle past the conflict point. The gap that the minor street vehicle accepts is shown as the headway between events 8 and 9, or 5.2 sec.

Figure 14 shows a time line for these events, covering the 30-sec period from t = 1:45 to t = 2:15. The minor street vehicle arrives at the stop line (Event 13 for movement 7) at t = 1:50.8 and departs (Event 14) at t = 1:54.2. Events 8 and 9 for movement 2 define the gap that is accepted by this vehicle. The headway corresponding to this gap is 5.2 sec.

We can further analyze these data by showing the number and size of gaps that are accepted and rejected. Figure 15 shows the frequency plot for gaps that are rejected, as represented by the headways for these gaps. We can note that with one exception, all drivers accept gaps with headways that are 5.5 sec or longer. Again, with one exception, all drivers reject gaps with headway that are 5.5 sec or shorter. While this is not a statistically sound method, we could assume that the critical headway in this case is about 5.5 sec.

a. minor street vehicle arrives at stop line

b. major street vehicle passes conflict point

c. minor street vehicle enters intersection

d. major street vehicle passes conflict point

Figure 13. Four events at example intersection

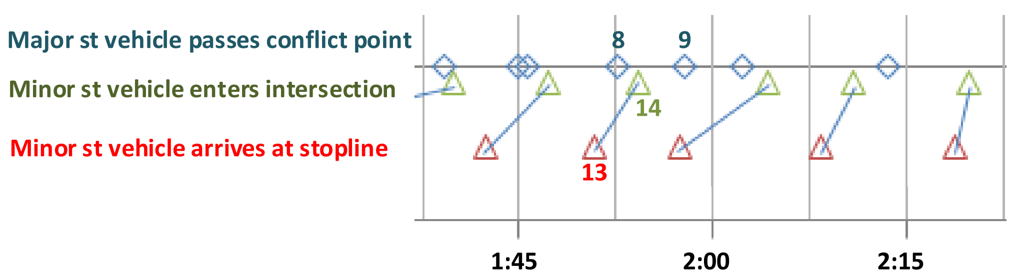

Figure 14. Time line for vehicle events

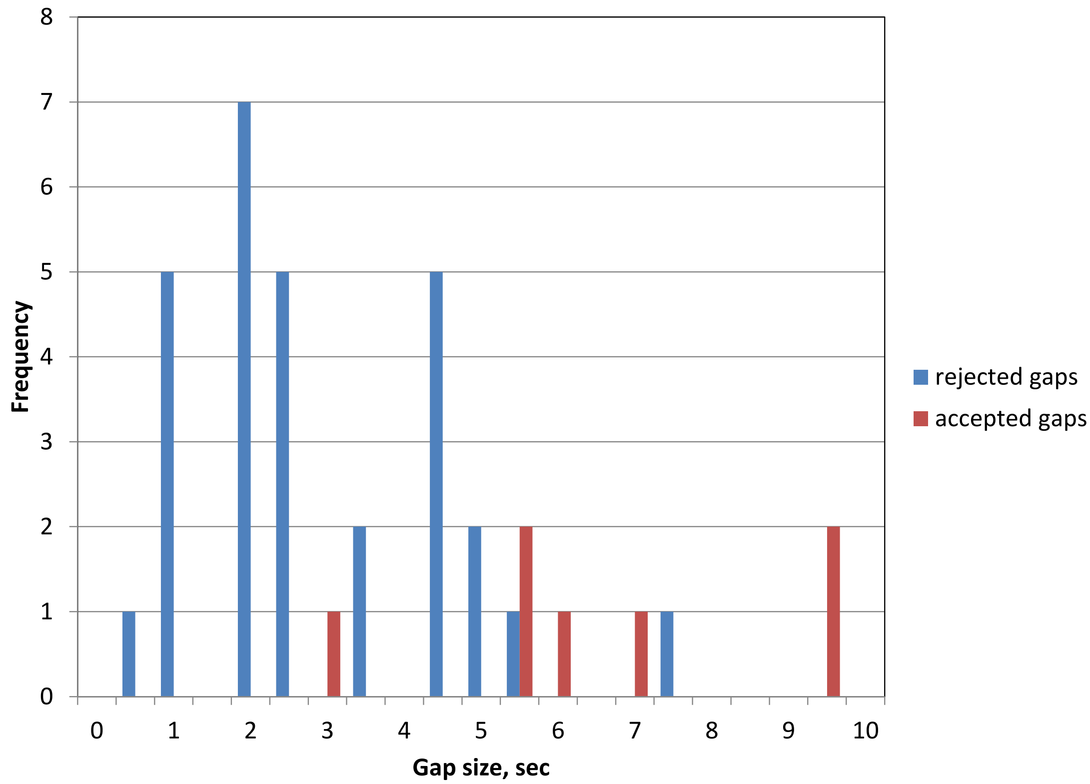

Figure 15. Accepted and rejected gap data

Calculating the Critical Headway

[Example calculation of tc example data, and Rod Troutbeck MLE process and spreadsheet.]

Example Calculation 6

[add]

Calculating the Follow Up Headway

[Example and calculation of follow up headway.]

Example Calculation 7

[add]

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7. Building a Computational Engine and Exploring the Model

Let's now set up a computational engine for each of the two simplified scenarios and explore what the models predict under a variety of traffic flow conditions.

Scenario 1

For Scenario 1, we assume an intersection that consists of two-one lane one-way streets. The capacity of the minor street approach is given by:

Equation 18

$$c = \frac{v_c e^{-v_c t_c/3600}}{1 - e^{-v_c t_f/3600}}$$

The computational engine should satisfy the following requirements:

- Accepts volumes (v, veh/hr) for the two approaches as inputs.
- Assumes the critical headway (t_c, sec) and follow up headway (t_f, sec) for the minor street movement.
- Computes the conflicting flow for the minor street movement.
- Computes the capacity for the minor street movement.

Figure 16 shows the computational engine template. The input volumes are entered in row 5. The critical headway and follow up headway for movement 8 are given in rows 6 and 7. The conflicting flow for movement 8 (NBTH movement) is calculated in row 10. The capacity for movement 8 is calculated in row 11.

	A	B	C	D
1	**TWSC Intersection Model (Scenario 2) - Computational Engine**			
2				
3	**Given Conditions**	**EBTH**	**NBTH**	
4	**Movement**	2	8	
5	Volume			
6	t_c		4.1	
7	t_f		2.2	
8				
9	**Calculations**	**EBTH**	**NBTH**	
10	Conflicting flow, v_c			
11	Capacity, c			

Figure 16.Computational engine template for TWSC intersecctions, Scenario 1

To assist you in constructing the computational engine, Table 9 shows the formulas for several of the cells.

Table 9. Example formulas for TWSC intersection computational engine

Cell	Formula
C10	=B5
C11	=PotCap(C10,C6,C7)

The code for the VBA function PotCap is shown in Table 10. The function requires the conflicting flow (v_c), the critical headway (t_c), and the follow-up headway (t_f) as inputs. The capacity calculation is performed in three steps. The numerator (term1) and denominator (term2) are calculated first and the capacity is then computed as term1 divided by term2.

Table 10. Function to calculate potential capacity for TWSC intersection computational engine

```
Public Function PotCap(vc, tc, tf)
'This function computes the potential capacity for a movement based on
vc, tc, and tf
term1 = Exp(-vc * tc / 3600)
term2 = 1 - Exp(-vc * tf / 3600)
PotCap = vc * (term1 / term2)

End Function
```

Scenario 2

For Scenario 2, we assume a T-intersection that consists of a major street with two-way traffic and a minor street with one movement.

The computational engine should satisfy the following requirements:

- Accepts volumes (v, veh/hr) for the three approaches as inputs.
- Assumes critical headway (t_c, sec) and follow up headway (t_f, sec) for movements 4 and 7.
- Computes the conflicting flow for movements 4 and 7.
- Computes the potential capacity for movements 4 and 7.
- Computes the impedance factor for movement 7
- Computes the movement capacity for movements 4 and 7.

Figure 17 shows the computational engine template. The input volumes are entered in row 6. The critical headway and follow-up headway for movements 4 and 7 are given in rows 7 and 8. The conflicting flow for movements 4 and 7 is calculated in row 11. The potential capacity for movements 4 and 7 are calculated in row 12. The impedance factor for movement 7 is calculated in row 13. The movement capacity for movements 4 and 7 is calculated in row 14.

	A	B	C	D
1	**TWSC Intersection Model (Scenario 2) - Computational Engine**			
2				
3	**Given Conditions**	**EBTH**	**WBLT**	**NBLT**
4	**Movement**	**2**	**4**	**7**
5	Rank	1	2	3
6	Volume			
7	t_c		4.1	7.1
8	t_f		2.2	3.5
9				
10	**Calculations**	**EBTH**	**WBLT**	**NBLT**
11	Conflicting flow, v_c			
12	Potential capacity, c_p			
13	Impedance factor			
14	Movement capacity, c_m			

Figure 17. Computational engine template for TWSC intersection, Scenario 2

To assist you in constructing the computational engine, Table 9 shows the formulas that are used for the NBLT movement.

Table 11. Example formulas for TWSC intersection computational engine

Cell	Formula
D11	=B6 + 2*C6
D12	=PotCap(D11,D7,D8)
D13	=1-C6/C12
D14	=D12*D13

The code for the Visual Basic function PotCap is shown, as before, in Table 10.

Example Calculation 8

Consider a TWSC intersection that includes two one-way streets, the configuration described earlier in this chapter as Scenario 1. Movement 2 has a volume of 600 veh/hr while movement 8 has a volume of 100 veh/hr. Figure 18 shows the resulting capacity for movement 8 using the computational engine.

	A	B	C	D
1	**TWSC Intersection Model (Scenario 1) - Computational Engine**			
2				
3	**Given Conditions**	**EBTH**	**NBTH**	
4	**Movement**	**2**	**8**	
5	Volume	600	100	
6	t_c		4.1	
7	t_f		2.2	
8				
9	**Calculations**	**EBTH**	**NBTH**	
10	Conflicting flow, v_c		600	
11	Capacity, c		987	

Figure 18. Example Calculation 8 for Scenario 1

Example Calculation 9

Let's now consider a TWSC T-intersection, the configuration described earlier as Scenario 2. The given data and results are given in Figure 19. Movement 4 (eastbound through vehicles) has a capacity of 987 veh/hr based on the conflicting flow of 600 veh/hr for movement 2 (westbound left turn vehicles). Movement 7 has a potential capacity of 306 veh/hr. But vehicles in movement 4 utilize some of this potential capacity as they wait to complete their left turn maneuvers. Specifically, for the calculated impedance factor of 0.899, movement 7 is blocked 10.1 percent of the time (1.000 – 0.899). The resulting movement capacity for movement 7 is 275 veh/hr.

	A	B	C	D
1	**TWSC Intersection Model (Scenario 2) - Computational Engine**			
2				
3	**Given Conditions**	**EBTH**	**WBLT**	**NBLT**
4	**Movement**	**2**	**4**	**7**
5	Rank	1	2	3
6	Volume	600	100	50
7	t_c		4.1	7.1
8	t_f		2.2	3.5
9				
10	**Calculations**	**EBTH**	**WBLT**	**NBLT**
11	Conflicting flow, v_c		600	800
12	Potential capacity, c_p		987	306
13	Impedance factor			0.899
14	Movement capacity, c_m			275

Figure 19. Example Calculation 9 results

8. Summary

In this chapter we explored the models on which the HCM capacity analysis method for TWSC intersections is based. We reviewed how a TWSC intersection operates in the field, and the factors on which models to predict the capacity of an intersection approach could be based. In developing the models we used the concept of the simplified scenario, in which only the most important traffic and geometric factors are considered. By focusing only on these factors, you developed a basic understanding of the operation of a TWSC intersection, one that can later be built on as the more complex conditions found in the real world are considered.

You studied two such scenarios. The first scenario is based on the intersection of two one-way streets, while the second scenario is based on a T-intersection, with three traffic streams or movements. In both scenarios, we assumed through movements only (no turning movements), a traffic stream consisting only of passenger cars, and no pedestrians to impede the flow of automobile traffic. You learned that the operation of a TWSC intersection is based primarily on the size of gaps in the major traffic stream that available to and usable by vehicles in the minor traffic stream. Each minor stream driver must decide whether a gap in major street traffic is large enough to enter safely. The higher the major street traffic volumes, the fewer and smaller are such usable gaps, and thus the lower the capacity of the minor traffic street.

9. Glossary

Table 12. Terms used in this chapter

Term	Definition
Critical headway	The minimum headway in the major street traffic stream required by minor stream vehicles to safely enter the intersection and complete their maneuver.
Follow up headway	The headway between vehicles in the minor traffic stream during conditions of continuous queuing using the same major stream gap.
Gap	The distance or time between the back bumper of one car and the front bumper of the following car in the major traffic stream.
Gap acceptance	The queuing model that describes how minor stream vehicles utilize gaps in the major street traffic stream.
Impedance	
Movement capacity	The capacity of a movement considering the traffic flow of higher priority traffic streams.
Potential capacity	The capacity of a movement regardless of the traffic flow of higher priority traffic streams
Rank	The relative priority of a traffic stream.

Table 13. Variables used in this chapter

Variable	Description	Units
c	Capacity	veh/hr
c_m	Movement capacity	veh/hr
c_p	Potential capacity	veh/hr
$p_{o,i}$	Probability of queue free state for movement i	-
t_c	Critical headway	sec
t_f	Follow up headway	sec
v	volume	veh/hr
v_c	Conflicting flow	veh/hr
λ	Flow rate	veh/sec

10. References

Werner Brilon wrote a comprehensive review of the work on the capacity of TWSC intersections as part of the 1989 symposium Intersections without Traffic Signals. Brilon and Rod Troutbeck provide a comprehensive review of the theory of unsignalized intersections as part of the Federal Highway Administration monograph on Traffic Flow Theory. Brilon, Troutbeck, and Marion Tracz provide a thorough review of the capacity of unsignalized intersections published in 1997. Karsten Baass wrote an excellent derivation of the capacity equation that is the basis for TWSC intersection capacity; this derivation is used in this chapter. Troutbeck developed a calculation process for calculating the critical headway that is covered in this chapter. Kyte and others developed critical headway and follow up headway values for TWSC intersections in the U.S. and used these values to calibrate a capacity model.

Werner Brilon, “Recent Developments in Calculation Methods for Unsignalized Intersections in West Germany,” in “Intersections Without Traffic Signals”, Werner Brilon, editor. Springer-Verlag, Berlin, 1989

Rod Troutbeck and Werner Brilon, “Theory of Unsignalized Intersections”, in Revised Monograph on Traffic Flow Theory, Federal Highway Administration. Web link: https://www.fhwa.dot.gov/publications/research/operations/tft/

Werner Brilon, Rod Troutbeck, and Marion Tracz. “Review of International Practices Used to Evaluate Unsignalized Intersections.” Transportation Research Circular Number 468. Transportation Research Board, April 1997.

Karsten Baass, “The Potential Capacity of Unsignalized Intersections.” ITE Journal, October 1987.

[Troutbeck citation]

Kyte, M., Z. Tian, Z. Mir, Z. Hameedmansoor, W. Kittelson, M. Vandehey, B. Robinson, W. Brilon, L. Bondzio, N. Wu, and R. Troutbeck. NCHRP Web Document 6: Capacity and Level of Service at Unsignalized Intersections: Final Report, Vol 1--Two-Way Stop-Controlled Intersections. Transportation Research Board, National Research Council, Washington, D.C., April 1996.